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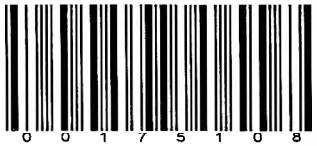

MEDICAL AND BIOLOGICAL PHYSICS

Laboratory works

Tsurganov A. G.

For foreign students of the first year

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The issue "Medical and biological physics. Laboratory works" is prepared in conformity with the typical syllabus for faculty of General Medicine. This issue contains all necessary data (text, formulas and tables) for study of medical and biological physics.
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INTRODUCTION

Development of the modern medicine is impossible without wide using of achievements of physics, technics, medical instrument making. The doctor should have not only necessary theoretical grounding, but also know the device and a principle of activity of applied equipment, to use the basic measuring instruments, to have the elementary skills of physical experiment, to be able to analyse and process the gained effects and to do the conforming deductions. The combination of these demands has defined selection of the laboratory works which have been included in the given physical practical work.

A series of works have original character, the others were repeatedly recasted and perfected by employees of our chair. For statement of laboratory works is used the standard educational equipment; some devices were released by the medical industry and made on chair. Each laboratory work contains the short theory on a studied question, the description of the laboratory devices and an operation procedure. At the end of each work are given control questions for students for preparation and protection of laboratory work. The practical work contains the necessary tabular data.

The laboratory practical work provides three basic steps of operation:

- studying of the theory of the explored phenomenon on short introduction to laboratory work that does not free students from necessity of studying of the conforming sections of the educational literature recommended to given work; preliminary stydying with the laboratory measuring instruments under its description in a practical work;
- execution of the work consisting in detailed studying of the laboratory apparatus on a workplace and conducting of necessary measurings;
- the report consisting in studying and processing of results, in the answer to the questions, including a procedure of laboratory work and the theory of the viewed phenomenon.

The present practical work urged to promote deeper understanding of a theoretical material, mastering by practical skills and formation of the scientific outlook of the future specialists.

PROCESSING OF RESULTS OF A DIRECT AND INDIRECT MEASUREMENTS

1. Measurement errors

Measurement is a determination of value of a physical quantity (for example: mass, length, velocity of blood flow, blood pressure, etc.) by practical consideration by means of hardware components.

Observed datas almost never give exact (true) value of quantity as they contain any errors. The measured value x_{meas} of some physical quantity x usually is differ from an ideal value x_{true}. The divergence of the value gained in experiment from the ideal value, i.e. a difference $|\tilde{o}_{true} - \tilde{o}_{meas}| = \Delta\tilde{o}$ is known **as an absolute error of measuring.**

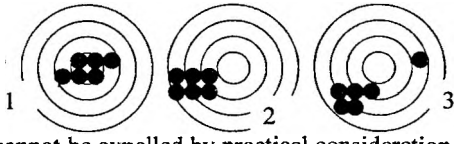
All errors of measurements are subdivided on **systematic, instrumental, random and gross errors.**

The systematic errors are errors, which magnitude and sign from experiment to experiment are maintained or change naturally (fig. 1, №2). They distort an observed data in one direction, either uprating, or underrating it. Similar errors cause constantly efficient causes, for example: restricted accuracy of the device or its derangement, inexact weight of set of weights, inexact marking of the scale of a ruler, delayed course of a stop watch, insufficient working out of experimental procedure.

It is impossible to reduce the contribution of systematic errors to observed datas by means of recurring of experiment. It can be made only by perfection of devices, improvement or modification of a measurement procedure, or introduction of any corrections in observed datas.

The random are call errors, which magnitudes and signs unexpected variate from experiment to experiment (fig. 1, №1). Such errors originate, for example, at weighting because of oscillations of the installation, unequal agency of a friction, temperature, at a rounding off of instrument readings. They can be related with measured object, for example, at measuring of the diameter of a cylinder having not absolutely round and unequal cross-section in various places. Random errors originate because of an imperfection or defect of organs of sense of the experimenter. At mathematical exposition in the theory of errors random errors of measurings are considered as random variables.

Fig. 1



Random errors cannot be expelled by practical consideration. Their influence on effect of measuring can be estimated by means of mathematical methods of

statistics. Decrease of influence of random errors is attained in experiment by multiple measurements.

Gross errors are the errors essential exceeding systematic and random errors. For example, they originate owing to inattention or tiredness of the experimenter, non-observance of appropriate requirements of the measurements, wrongly digitized or noted numerals, application of the unworkable device. The observations keeping gross errors are rejected as uncertain. Obvious representation of a relation between random (1), systematic (2) and gross errors (3) in case of shooting on targets is shown on fig. 1.

Instrumental errors are the errors originating because of an imperfection of a measuring apparatuses. These errors are defined by the grade accuracy of devices and cannot be completely expelled at measurings.

Definition of errors of measuring instruments is viewed below.

The relation of the instrumental and random errors defins a number of necessary measurings. We will view two cases.

1. Let at repeated measurements of some quantity each time is gained the same value. The random error in the given method of measurement plays a supporting role that speaks about small test-sensitivity, and defining is error brought by the measuring instrument. Thus it is not meaningful to spend measurings more than once (one series) as nonsingle measurings reduce random, but not an instrumental error.

2. If at repeated measurements of some quantity are resulted values a little different from each other, the random error in this method of measurements is more than instrumental error. Thus it is necessary to repeate measurements some times to lower random errors. It is desirable to take number of measurements such that the random error of $\bar{x}$ of a measurand became less than the instrumental error.

2. Processing of data of direct measurements

Direct measurement is a measurement at which value of quantity interesting the experimenter are found directly from a reading of a device. **The indirect** are called measurements at which value of any quantity is found as function of other quantities. For example, resistance R is determined on voltage U and a current I.

Let it is executed n direct measurements x_1, x_2, ..., x_n of some quantity x, which true value x is marked out as x_{true}.

1. Estimation of the true value of quantity x_{true} is the **mean** of n measurements which is computed by formula:

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i .$$

2. For estimation of dispersion of separate values x_i concerning the mean $\bar{x}$ is calculated **standard deviation** S:

$$S = \sqrt{\frac{(\bar{x} - x_1)^2 + (\bar{x} - x_2)^2 + ... + (\bar{x} - x_n)^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} (\bar{x} - x_i)^2}{n-1}},$$

where $|\bar{x} - x_i| = \Delta x_i$ is absolute error of individual measurement. S will variate from one sample to another and it tends to some constant value σ, which it is possible to term as statistical limit of S: $\sigma = \lim_{n \to \infty} S$. The square of this quantity is known **as a variance S^2= D**. S is an approximate value, which is the more close to σ, than is more n.

It is necessary to distinguish S from $S_{\bar{x}}$, where $S_{\bar{x}}$ is standard error of mean: $S_{\bar{x}} = \frac{S}{\sqrt{n}}$. $S_{\bar{x}}$ characterises dispersion of mean $\bar{x}$ from sample to sample, but not variability of data in the given sample, which usually is interesting to the explorer.

3. For exclusion of gross errors which can strongly distort result is used a series of criteria. **3σ criterion** is used, if number of measurements n ≥ 20: if the difference between a prospective gross error and a mean modulo is more of 3σ, that is $|x_i - \bar{x}| > 3\sigma$, that it is improbable also and it is possible to reject it (Instead of σ in the formula is substituted S). **Romanovsky criterion** is used if n < 20. Compute the module of ratio $\left|\frac{x_i - \bar{x}}{\sigma}\right| = \beta$, where x_i is possible gross error, $\bar{x}$ is the mean *without value* of this gross error. The available value β compare to the table β_T at the chosen significance level α under the next table, and if $\beta \geq \beta_T$ than the checked value must be rejected.

Significance level α	Number of measurements n						
	4	6	8	10	12	15	20
0.01	1.73	2,16	2,43	2,62	2,75	2,90	3,08
0.02	1.72	2,13	2,37	2,54	2,66	2,80	2,96
0.05	1.71	2,10	2,27	2,41	2,52	2,64	2,78
0.1	1.69	2,00	2,17	2,29	2,39	2,49	2,62

4. We count an absolute (random) error Δx

$$\Delta x = t_{\alpha,n} \cdot S,$$

where $t_{\alpha,n}$ is Student coefficient (we determine it under the table) for number of measurements n and a confidence probability of p. *The confidence probability* is a probability that the deviation of the result received by us from value of a measurand does not exceed Δx. Confidence probability p in the conditions of educational laboratory is often equal to 0.95 or 95 %. It means that at multiple repeated of experiment at the same requirements the absolute error in 95 cases from 100 will not exceed value of Δx.

5. The result of measurements is noted as: $x = \bar{x} \pm \Delta x$, or $\bar{x} - \Delta x \leq x \leq \bar{x} + \Delta x$.

6. Finde a systematic instrumental error Δ_c. For example, the systematic error of a stop watch with scale interval of 0.2 s is equal to half of scale interval of the device, that is 0.1 s; the systematic error of a ruler scale is equal too half of scale interval, that is 0.5 mm, etc.
7. We compare a systematic error Δ_c and a random error Δx. If one of them in 3 times and more exceeds another, confidence interval limits are formed on a larger error: $x = \overline{x} \pm \Delta x$ or $x = \overline{x} \pm \Delta_c$. If errors are approximately equal, the confidence interval will become $x = \overline{x} \pm \delta$, where

$$\delta = \Delta_c + 2\Delta x$$

8. Relative error E is determined by formula:

$$E = \frac{\Delta x}{\overline{x}} \cdot 100\%.$$

Let's consider an example of processing of results of direct measurings.
Example. By stop watch had been made 5 measurements of time periods: 89.6; 89.2; 89.4; 89.0; 89.5. Give an interval estimation, if a significance level is α = 0,05 (p = 1- α = 0.95).
1. Mean:

$$\overline{x} = \frac{89,6 + 89,2 + ... + 89,5}{5} = 89,34 \approx 89,3 \text{ (s)}.$$

2. We compute standard deviation S:

$$S = \sqrt{\frac{(\overline{x} - x_1)^2 + (\overline{x} - x_2)^2 + ... + (\overline{x} - x_n)^2}{n-1}} = 0.3 \text{ (s)}.$$

3. Let's check the first measurement 89.6 on a gross error by Romanovsky test:
$\left|\frac{x_i - \overline{x}}{\sigma}\right| = \beta = 1$, $\beta_т$ (n=4, α=0.05) = 1.71 > 1, that is the gross error in this case is not present. It is analogously possible to check result 89.0 s.
4. We determine an absolute error $\Delta x = t_{\alpha,n} \cdot S = 2.8 \cdot 0.3 = 0.84 \approx 0.8$ (s), where Student coefficient is equal to $t_{\alpha,n} = 2.8$ (from the table).
5. Interval estimation: $x = \overline{x} \pm \Delta x$ =89.3±0.8 (s).
6. The systematic error Δ_c of the stop watch is equal to half of scale interval of the device (the scale division value = 0.2): Δ_c=0.2/2 = 0.1 (s).
7. We compare a systematic error Δ_c and a random error Δx: 0.1 and 0.8 s. Random error more than three times is more systematic error and by the systematic it is possible to neglect.
8. Finally, interval estimation: $x = \overline{x} \pm \Delta x$ =89.3±0.8 s (α = 0.05).

3. Processing of results of indirect measurements

If the unknown quantity y is connected with several quantities x_i by a certain functional connection:

$$y = f(x_1, x_2, x_3 ... x_n),$$

this measurement of y is termed *indirect* and estimation of its true value is the mean:

$$\bar{y} = f(\bar{x}_1, \bar{x}_2, ..., \bar{x}_n),$$

where $\bar{x}_1, \bar{x}_2, ...\bar{x}_n$ are means of values of directly measured quantities x_1, x_2, ..., x_n. Estimation of dispersion of the data round of mean in several samples is the standard error of mean of $\bar{y}$, computed by formula:

$$S_{\bar{y}} = \sqrt{\left(\frac{\partial f}{\partial x_1} S_{\bar{x}_1}\right)^2 + \left(\frac{\partial f}{\partial x_2} S_{\bar{x}_2}\right)^2 + ... + \left(\frac{\partial f}{\partial x_n} S_{\bar{x}_n}\right)^2},$$

where partial derivatives are computed for a mean $\bar{x}_i$; $S_{\bar{x}_i}$ is computed by formula the standard error of mean of direct measurement.

Interval estimation of y is the confidence interval in which hits the true value of indirectly measured quantity with the given confidence probability α:

$$\bar{y} - \Delta\bar{y} \le y \le \bar{y} + \Delta\bar{y}$$

For its determination is calculated an absolute error of mean $\bar{y}$:

$$\Delta\bar{y} = t_{\alpha,n} \cdot S_{\bar{y}}.$$

The result of measurement is noted as:

$$y = \bar{y} \pm \Delta\bar{y}$$

or in a following form that is ***more interesting*** to the researcher as gives an interval estimation in the *given* sample

$$\boxed{y = \bar{y} \pm \Delta y}, \text{ where } \Delta y = t_{\alpha,n} \cdot S_{\bar{y}} \cdot n$$

Relative error is $E = \frac{\Delta\bar{y}}{\bar{y}} \cdot 100\%$.

Let's consider an example of processing of results of indirect measurements.

Example. At *Emf=E* definition by a compensatory method on a design formula $E = E_0 \frac{l}{l_0}$ (where E is emf of an unknown source, E_0=1,018 V is a constant, l and l_0 are directly measurands (length of shoulders of the slide wire)) are lead three measurements of l= 4160, 4150, 4155 (mm) and l_0 = 3390, 3395, 3380 (mm). Give an interval estimation of emf E of an unknown source.

1. We calculate means of directly measurands l and l_0:

$$\bar{l} = \frac{4160+4150+4155}{3} = 4155 \ mm, \quad \bar{l}_0 = \frac{3390+3395+3380}{3} = 3388 \ mm.$$

2. We calculate the mean of indirectly determined quantity:

$$\bar{E} = E_0 \frac{\bar{l}}{\bar{l}_0} = 1.018 \cdot \frac{4155}{3388} = 1.25 \text{ V}.$$

3. We compute the standard deviations of $\bar{l}$ and $\bar{l}_0$:

$$S_{\bar{l}} = \sqrt{\frac{(\bar{l}-l_1)^2 + (\bar{l}-l_2)^2 + (\bar{l}-l_3)^2}{n(n-1)}} = 2880 \ mm,$$

$$S_{\bar{l}_0} = \sqrt{\frac{(\bar{l}_0-l_{01})^2 + (\bar{l}_0-l_{02})^2 + (\bar{l}_0-l_{03})^2}{n(n-1)}} = 4415 \ mm.$$

4. We find the partial derivatives

$\frac{\partial E}{\partial I} = (E_0 \frac{\bar{I}}{\bar{I}_0})'_I = \frac{E_0}{E}$; $\frac{\partial E}{\partial I_0} = (E_0 \frac{\bar{I}}{\bar{I}_0})'_{I_0} = \frac{E_0 \bar{I}}{\bar{I}_0^2}$.

5. We compute the standard error of mean $S_{\bar{E}}$ of indirectly defined quantity E:

$$S_{\bar{E}} = \sqrt{(\frac{\partial E}{\partial I_0} \cdot S_{I_0})^2 + (\frac{\partial E}{\partial I} \cdot S_I)^2} = \sqrt{(\frac{E_0 \bar{I}}{\bar{I}_0^2} S_{I_0})^2 + (\frac{\bar{E}_0}{\bar{I}_0} \cdot S_{I_0})^2} = 1.84 \cdot 10^{-3} \text{ V}.$$

6. Under tables we find the Student coefficient $t_{0.95;3} = 4.30$ and then emf absolute error:

$\Delta\bar{E} = S_{\bar{E}} \cdot t = 7.9 \cdot 10^{-3}$ V, $\Delta E = S_{\bar{E}} \cdot t \cdot \sqrt{n} = 13.7 \cdot 10^{-3} \approx 0.01$ V.

7. Unknown interval: $E = \bar{E} \pm \Delta E = 1.25 \pm 0.01$ V.

8. Relative error: $\varepsilon = \frac{\Delta E}{\bar{E}} \cdot 100\,\% = 1\%$.

Let's view some special cases.

1. The unknown quantity is the sum or a difference of two independent random quantities $y = x_1 \pm x_2$. Then the absolute error of $\bar{y}$ is equal $\Delta\bar{y} = \sqrt{\Delta\bar{x}_1^2 + \Delta\bar{x}_1^2}$ and $\bar{y} = \bar{x}_1 \pm \bar{x}_2$.
2. The unknown quantity is product of several independently measurands $y = x_1 \cdot x_2$. Then the absolute error of $\bar{y}$ is equal $\Delta\bar{y} = \sqrt{x_1 \cdot \Delta\bar{x}_2 + x_2 \Delta\bar{x}_1}$, $\bar{y} = \bar{x}_1 \cdot \bar{x}_2$, Relative error $\varepsilon = \frac{\Delta\bar{y}}{y} = \sqrt{(\frac{\Delta\bar{x}_1}{x_1})^2 + (\frac{\Delta\bar{x}_2}{x_2})^2}$.

GRAPHICAL FORM OF OBSERVED DATA

At processing of observed data in medical researches is used a graphical form of functional connections between investigated quantities.

At construction of graph it is necessary to be guided by following rules:

- on the abscissa axis to use an independent variable (argument), and on the axis of ordinates to use a dependent variable (function) which is an object of research. On each axis it is necessary to specify conditional letter designations of quantity and unity of its measurement (fig. 2);
- scales on axes can be any; **it is not necessary to specify a numerical value of measured quantity on axes;**
- the point of origin and scales on axes should be chosen so that all square of a paper restricted to axes has been used for plotting of graph. For this purpose the count of quantities on axes should begin not necessarily with null, but from the number, close to the least value of argument (function) and to end the greatest value of the measured quantities;
- the scale division value for axes should be not less absolute error of viewed quantities, differently precision under a graph will exceed precision of value of quantities superimposed on it, that is not meaningful;

– values of the observed datas scored on the graph by points are at centres of rectangles with the sides, equal to the doubled errors of argument $2\Delta x$ and function $2\Delta y$ (fig. 2). The line of graph should transit through fields of these rectangles and at enough of the data the line of graph should be smooth. The number of points on one and other side from a curve should be approximately equal. If the point is away from other points, experiment should be iterated some times at value of argument, close to value in this point. Thus it can appear that we deal with a gross error (then this point do not consider) or any regularity between studied quantities occurs.

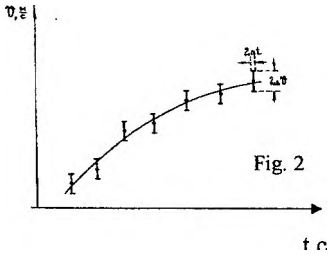


Fig. 2

Under a graph for value of argument which directly was not observed, it is possible to discover value of function, such method is known *as interpolation. The* method of determination of value of explored quantity outside of the line of graph by prolongation of a line is termed *as extrapolation.* We use extrapolation if a curve of graph is smooth and in explored dependence there are no bases to expect sharp modifications.

At realisation of laboratory works sometimes there is a necessity of making *of the calibration plot.* For its making it is desirable to choose the dependence expressed *by a straight line* because using by it will be the most convenient. For straight line deriving often use the *semilogarithmic scale*: it is the right-angled system, on which one axis is the uniform scale, and on another – logarithmic. The logarithmic scale is used for a graphical form of the quantities which value vary in a wide limits (for example, value of sound wave intensity).

CALCULATION ACCURACY

It is often compute the unknown quantity to five and more significant digits. This accuracy is excessive. Numerical value of result should not contain larger number of numerals, than the number set with the least accuracy. For improvement of value of last nonzero digit of result it is necessary to use a following rule: if it <5 it should be thrown, and if it ≥ 5 it is necessary to increment the previous numeral on 1.

It is not necessary to make calculations of errors of measurements to a closer approximation, than calculation of value of the measurement quantity. Examples of a definitive recording of observed datas:

Correctly	**Incorrectly**
284 ± 1	284.5 ± 1
2740 ± 12	2742 ± 12
350 ± 38	353 ± 38
52.7 ± 0.3	52.74 ± 0.3
13.840 ± 0.013	13.8372 ± 0.13

4.750 ± 0.006 4.75 ± 0.0006

ESTIMATION OF ERRORS OF ELECTRICAL MEASUREMENTS

Accuracy of the measuring instrument is the least value of quantity which can be determined reliable by means of the given device. Accuracy of the device is specified on the device or in the certificate. *If accuracy of the device is unknown,* it is defined **by half of scale interval of the least scale division** of the device (ruler, stop watch, thermometer).

On the scale of the electrical device is superimposed the symbol specifying a type of a measured quantity: ampermeter (A), voltmeter (V), ohmmeter (Ω); current type: direct (–), alternating (~); position at measurement: vertical (↑), horizontal (→); grade of accuracy (it is sometimes encircled by a circle).

Grade of accuracy is the ratio of an absolute error Δx to the greatest value of a measured quantity $x_н$ expressed in percentage ($x_н$ is the nominal value): $\gamma_н = \frac{\Delta x}{x_н} \cdot 100\,\%$ (1). From the formula (1) follows: $\Delta x = \gamma_н \cdot \frac{x_н}{100}$ (2). Generally the error can be positive or negative.

Relative error of any quantity measured by the electrodevice is called the ratio of the greatest possible absolute error Δx of the device to the measured value of this quantity x_t expressed in percentage:

$$E = \frac{\Delta x}{x_t} \cdot 100\,\% \quad (3).$$

Example. There is a voltmeter, which grade of accuracy is $\gamma_i = 1.5$ and limit of measuring is $U_н = 40$ V.

Device absolute error:

$$\Delta U = 1.5 \cdot 40/100 = 0.6\ (\text{V}).$$

Let's suppose that by means of the device two value of a voltage are measured: $U_1 = 10$ V and $U_2 = 30$ V. Let's compute for each case a relative error:

$$E_1 = \frac{\Delta U}{U_1} \cdot 100\,\% = \frac{0.6 \cdot 100\,\%}{10} = 6\,\%,\ E_1 = \frac{\Delta U}{U_2} \cdot 100\,\% = \frac{0.6 \cdot 100\,\%}{30} = 2\,\%.$$

From the example follows that the less value of reading on the device is the more relative error. Therefore it is recommended to choose devices that *readings were made more close to the scale end.*

SAFETY REQUIREMENTS IN EDUCATIONAL LABORATORY

1. Before execution of laboratory works students should have instructing in safety measures.

2. At finding of faultiness of the device it should be switched off immediately. It is forbidden to pluck a plug by the cord, only by means of the plug case.
3. Completed electric circuit can be switched on only after checkout by the teacher, and upon termination of work all electrical circuits should be switched off.
4. Parts of electrical devices accessible to touch should be grounded.
5. At operation with the UHF and SHF generators for decrease of radiation in ambient space the voltage should be given only after installation of ultrasonic head or radiator in the necessary position.
6. Laser radiation should not act on eyes.

Laboratory work №1

Methods of mathematical statistics for processing of a medical and biological information

Purpose of the work: study of statistical methods of processing of the skilled data submitting to the normal law of distribution; construction of the histogram and Gaussian curve.

Equipment: special scale for measurement of distance between the centers of eyes.

Theory of the work

Random variable is called a variable that accepts values depending from random circumstances, for example: life expectancy of a person, a pulse rate etc. **Discrete** random variable accepts values separately from each other, for example: number of patients on reception at a doctor. Values of a **continuous** random variable are fill any interval, for example: temperature of the patient, systolic pressure …

Let in n tests the random variable x_i has met m_i times and then $P_i^* = \frac{m_i}{n}$ is called *relative frequency*. At the big number of tests P_i^* tends to probability of occurrence of value x_i^* : $\lim_{n\to\infty} \frac{m_i}{n} = P_i$.

Numerical characteristics of random variables are:

1). Mean μ or M(x); for a discrete random variable mean is

$$\mu = x_1 \frac{m_1}{n} + x_2 \frac{m_2}{n} + \ldots + x_n \frac{m_n}{n} = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n = \sum_{i=1}^{n} x_i p_i \quad \text{or} \quad \mu = \sum_{i=1}^{n} x_i p_i$$

At big number of measurements (tests) μ is equal to *average arithmetic value* $\bar{x}$.

2). **Variance D** characterizes dispersion of a random variable around of its mean:

$$D = \sum_{i=1}^{n} (x_i - \mu)^2 p_i .$$

For estimation of dispersion of a random variable in terms of the *same dimension* as x_i is entered concept of **standard deviation σ**: $\sigma = \sqrt{D}$. For small sample (n <30) the formula $s^2 = \frac{n}{n-1} D$ gives the best estimation of dispersion.

Law of distribution of a random variable is the parity establishing connection between its possible values and their probabilities. The law of distribution can be written down by several ways: 1) tabulared for discrete variables; 2) graphic: as the histogram for discrete variables and as a curve of distribution for continuous variables.

There are some laws of distribution of random variables (binominal, Poisson distribution etc.). The normal law of distribution or law of Gauss takes the major place in medical statistics: distribution of height, pressure, weight of people and other parameters (20%) submits to **law of Gauss**:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where x is value of a random variable, f(x) is density of probability, μ is mean, σ is standard deviation.

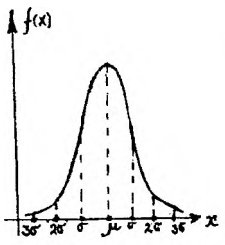


Fig.1

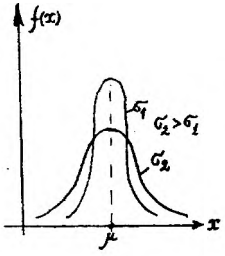


Fig. 2

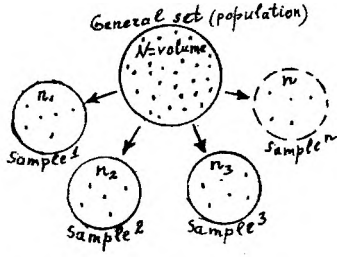


Fig. 3

Diagram of normal Gauss law is on the fig. 1. **Properties of Gaussian curve:**

1) a curve is symmetric relatively of x=μ and at $x \to \pm\infty$, $f(x) \to 0$;

2) form of a curve depends on σ (fig. 2); 3) areas under all curves are equal to 1;

4) area under a curve between ordinates $\mu\pm\sigma$ makes approximately 68 % from the area under all curve. It means that approximately 68 % of all random variables deviates from μ no more than on σ. The area under a curve between ordinates $\mu\pm2\sigma$ is approximately 95% and under the $\mu\pm3\sigma$ approximately 99.7%, i.e. practically all random variables will be in the interval of $\pm3\sigma$ **(law of «3σ»)**.

All analyzed objects forms **general set (population)**, for example: at study of distribution of distance of centre to centre of eyes general set is all students of university. The number of objects N of general set is known as its **volume N**. It is impossible to examining all objects of general set, therefore for their study choose a part of objects or **sample** of volume n, for example, at us sample is students of any group (fig.3). Values of a random variable of any sample forms **simple statistical series**; if to arrange random variables in ascending order we shall receive a **variational series**. For convenience of the analysis all range of random variables we can divide on *k* intervals (formula of **Stergess**):

$$k = 1 + 3.33 \lg n$$

where n is volume of a sample. Then count up number of random variables m_i including in each interval (frequencies) or relative frequencies $P_i^* = \frac{m_i}{n}$. The table containing x_i, m_i or p_i^* is named a **statistical interval series of distribution** and its graphic shows the **histogram** (fig. 3).

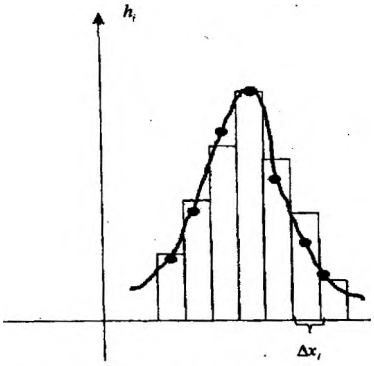


Fig 4

Histogram is a set of rectangles with height of $h_i = \frac{P_i^*}{\Delta x_i}$ and with the basis of Δx_i. The area of each rectangle is equal to P_i^* and area of all histogram is equal to 1.

If the random variables are distributed under the normal law then with increase of number of intervals the histogram will nearer more and more to curve of Gauss

(fig. 4). The problem of selection of a curve in the given statistical series is known as a *problem of alignment of series.*

One of the methods that used for alignment of series is that parameters μ and σ of Gauss law take equal to parameters of μ^* and σ^* of variational series of sample:

$\mu=\mu^*$, $\sigma=\sigma^*$, where $\mu^* = \sum_{i=1}^{k} \bar{x}_i P_i^*$ (1)

is mean of variational series,

$$\bar{x}_i = \frac{x_1 + x_2 + \ldots + x_{m_i}}{m_i} \quad (2)$$

is average arithmetic value for each interval of variational series,

$$\sigma^* = \sqrt{\sum_{i=1}^{k} (\bar{x}_i - \mu^*)^2 p_i^*} \quad (3)$$

is standard deviation of variational series.

Let's put $\bar{x}_i$, μ^* and σ^* to the Gauss law

$$f(\bar{x}_i) = \frac{1}{\sigma^* \sqrt{2\pi}} \cdot e^{-\frac{(\bar{x}_i - \mu^*)^2}{2\sigma^{*2}}} \quad (4)$$

For simplification of calculations under the formula (4) in the beginning we shall find value Z_i

$$Z_i = \frac{|\bar{x}_i - \mu^*|}{\sigma^*}, \quad \text{then } f(\bar{x}_i) = \frac{1}{\sigma^* \sqrt{2\pi}} e^{-\frac{1}{2} Z_i^2} \quad (5)$$

Let's allocate function $\varphi(Z_i) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} Z_i^2}$, then $f(\bar{x}_i) = \frac{\varphi(Z_i)}{\sigma^*}$ (6)

Value of $\varphi(Z_i)$ can be founded from the tables.

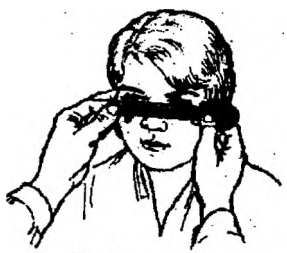


Fig. 5

In the given work **as random variable X** is used **a distance between centre to centre of pupils of eyes x_i.** At assignment of glasses is very important not only to select lenses correctly, but also correctly to measure x_i (eye base) for a long work without effort (fig. 5).

So *we shall study the law of distribution of the*

random variable x_i (distance between centre to centre of pupils of eyes).

Order of the work

1. With help of the special scale are measured distances x_i between centre to centre of pupils at 100 students (men and women) and brought in the table 1 in ascending order, i.e. is made a variational series of values x_i (mm):

Table 1

№	x	№	x	№	x	№	x	№	x	№	x	№	x	№	x	№	x	№	x
1	56	**11**	58	**21**	59	**31**	60	**41**	61	**51**	62	**61**	62	**71**	63	**81**	64	**91**	64
2	56	**12**	58	**22**	59	**32**	60	**42**	61	**52**	62	**62**	62	**72**	63	**82**	64	**92**	65
3	56	**13**	58	**23**	59	**33**	60	**43**	61	**53**	62	**63**	62	**73**	63	**83**	64	**93**	65
4	57	**14**	58	**24**	59	**34**	60	**44**	61	**54**	62	**64**	62	**74**	63	**84**	64	**94**	65
5	57	**15**	58	**25**	60	**35**	60	**45**	61	**55**	62	**65**	62	**75**	63	**85**	64	**95**	65
6	57	**16**	58	**26**	60	**36**	60	**46**	61	**56**	62	**66**	62	**76**	63	**86**	64	**96**	65
7	57	**17**	59	**27**	60	**37**	61	**47**	61	**57**	62	**67**	62	**77**	63	**87**	64	**97**	66
8	57	**18**	59	**28**	60	**38**	61	**48**	61	**58**	62	**68**	62	**78**	63	**88**	64	**98**	66
9	57	**19**	59	**29**	60	**39**	61	**49**	61	**59**	62	**69**	62	**79**	63	**89**	64	**99**	67
10	58	**20**	59	**30**	60	**40**	61	**50**	62	**60**	62	**70**	62	**80**	64	**90**	64	**100**	68

2. For plotting of the histogram we construct statistical interval series of distribution:

a) we divide all range of values x_i on $k = 1+3.3\lg 100 \approx 7$ intervals;

width of an interval is $\Delta x = \frac{x_{max} - x_{min}}{k} = \frac{69-55}{7} = 2$ mm.

The data is brought in the 1st line of the table 2;

b) we calculate number m_i of values x_i that got in each interval (2nd line);

c) with help of the formula $P_i^* = \frac{m_i}{100}$ we determine relative frequencies P_i^* for each interval (3d line).

3. The histogram is the graphic representation of statistical interval series of

distribution. For construction of the histogram we count up values of heights $h_i = \frac{P_i^*}{\Delta x_i}$ of each rectangular. On each interval we build a rectangular of height h_i.

4. With help of formulas (2), (1) and (3) we calculate $\overline{x_i}$, μ^* and σ^*:

$\overline{x_1} = \frac{56+56+56}{3} = 56;\ \overline{x_2} = \frac{57+57+57+57+57+57+58+58+58+58+58+58+58}{13};$

$\overline{x_3} = 59.6;\ \overline{x_4} = 61.6;\ \overline{x_5} = 63.5;\ \overline{x_6} = 65.3;\ \overline{x_7} = 67.5.$

$\mu^* = 56 \cdot 0.03 + 57.5 \cdot 0.13 + 59.6 \cdot 0.20 + 61.6 \cdot 0.34 + 63.5 \cdot 0.21 + 65.3 \cdot 0.07 + 67.5 \cdot 0.02 = 61.5$

$\sigma^* = \sqrt{(56-61.5)^2 \cdot 0.03 + (57.5-61.5)^2 \cdot 0.13 + (59.6-61.5)^2 \cdot 0.2 + (61.6-61.5)^2 \cdot 0.34 +}$

$\sqrt{63.5-61.5)^2 \cdot 0.21 + (65.3-61.5)^2 \cdot 0.07 + (67.5-61.5)^2 \cdot 0.01} = 2.43.$

5. We calculate values Z_i with help of formula (5) for each interval (line 6).

6. We find with help of the table value $\varphi(Z_i)$ for each interval (line 7).

7. We calculate for each interval values of function of distribution $f(\overline{x_i})$ using the formula (6) (line 8).

Table 2

1	Δx_i	55-57	57-59	59-61	61-63	63-65	65-67	67-69
2	m_i	3	13	20	34	21	7	2
3	$P_i^* = \frac{m_i}{n}$	0.03	0.13	0.20	0.34	0.21	0.07	0.02
4	$h_i = \frac{P_i^*}{\Delta x_i}$	0.015	0.065	0.10	0.17	0.105	0.035	0.01
5	$\overline{x_i}$	**56**	**57.5**	**59.6**	**61.6**	**63.5**	**65.3**	**67.5**
6	Z_i	2.26	1.65	0.78	0.04	0.82	1.56	2.47
7	$\varphi(Z_i)$	0.031	0.1023	0.2943	0.3986	0.285	0.1182	0.0189
8	$f(\overline{x_i})$	**0.012**	**0.042**	**0.121**	**0.164**	**0.117**	**0.049**	**0.008**

8. According to lines 5 and 8 we build the graph of function $f(\overline{x_i})$ in the same system of coordinates with our histogram (fig. 6):

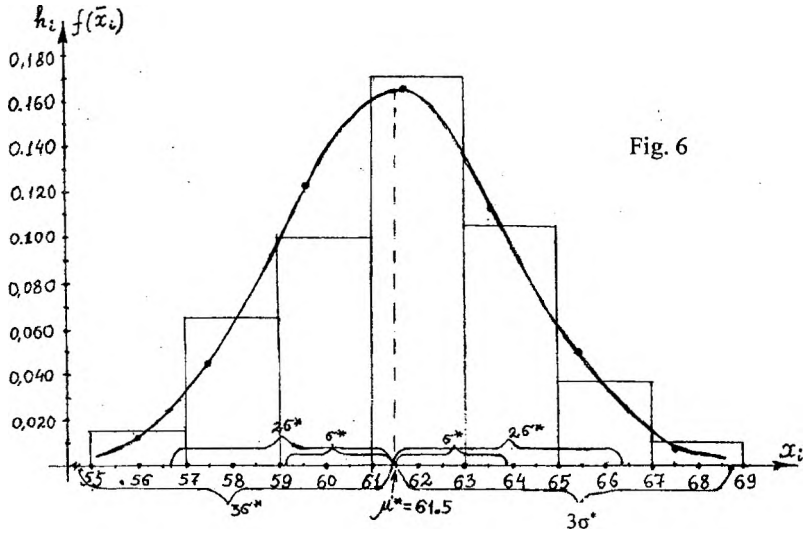


Fig. 6

Conclusion:

a) the area under the histogram is approximately equal to the area under the curve of distribution;

b) the curve has symmetric bell-shape form;

c) in intervals of $\mu \pm \sigma$, $\mu \pm 2\sigma$ $\mu \pm 3\sigma$ gets accordingly ≈ 68%, 96% and 100% measurements, the rule of the «three σ» is carried out.

It is possible to draw the qualitative conclusion: **distribution of distance of centre to centre pupils of eyes is described by normal distribution.**

Control questions

1. Write down numerical characteristics of random variables.
2. Write down law of Gauss; decipher the values which are included in the formula of the law; properties of the Gaussian curve.
3. What is the statistical interval series of distribution?
4. What is the histogram? How does it construct?
5. Determination of μ and σ for Gaussian curve.

Laboratory work №2

Testing of statistical hypotheses

Purpose of the work: to familiarize with the basic concepts of statistical testing of hypotheses; to consider using of criteria of Student, Fisher, Wilcoxon-Mann-Whitney.
Equipment: tables of corresponding criteria.

Theory of the work

Why statistical methods are used in medicine?

Development of ideas of a critical estimation of the medical information has resulted in occurrence in 80th years of the last century of the concept of **Evidence Based Medicine** (EBM). Basic positions of EBM are: 1) each decision of the doctor should be based on the scientific data; 2) the weight of each fact is more, than more strictly a technique of research during which this fact is received. As "The gold standard" are considered randomized (i.e. received as result of casual selection) controllable researches. Individual medical experience and opinion of experts or "authorities" are considered as not having a sufficient scientific basis.

One of the major components of EBM is use of the scientifically-grounded statistical methods, one of which is check of statistical hypotheses.

Concept of a statistical hypothesis.

The statistical hypothesis (H) is any assumption about *a kind* or *parameters* of population which is checked on the basis of the sample data.

For example: 1) **H:** the weight of newborns is distributed under the normal law (hypothesis about a kind of distribution);

2) **H:** average values ($M_{(Y)} = M_{(X)}$) of arterial pressure in two groups of patients are equal, i.e. both samples are taken from the same population (hypothesis about parameters of distribution).

Testing of statistical hypotheses are connected with estimation of various processes: comparison of medical techniques, characteristics of preparations and medical equipment, efficiency of treatment, duration of illness, profitability, etc.

Zero and alternative hypotheses.

Tested hypothesis is called *null* and designated by H_0. The null hypothesis always rejects effect of interference. Alongside with the null hypothesis also is considered one of the *alternative* hypotheses that is designated as H_1.

For example, let a lot of a pharmaceutical medicine supervise on small sample and compare to norm; then null hypothesis H_0: the released production lot of pharmaceutical medicine is non-standard (spoilage) and alternative is H_1: the lot corresponds to norm.

Problem of testing of statistical hypotheses

The problem of test of hypotheses: on the basis of the analysis of the sample data (the incomplete information) to make the decision on validity of one of hypotheses.

Type I and type II errors. Significance level α.

At testing of hypotheses because of presence of the incomplete information two types error can occur (see the table):

1. $H_1$is accepted, when true is H_0; that is H_0 wrongly rejected: **the type I error** (the false conclusion about existence of distinctions which really are not present, or "hyperdiagnostics").

2. H_0 is accepted, when is true H_1: **the type II error** (to not find really existing distinctions, or "hypodiagnostics").

A type I error is an alarm without the fire and a type II error is a fire without an alarm.

The result received at testing ↓	*That is fact*	
	Hypothesis Ho is true	Hypothesis H_1 is true
Hypothesis Ho is accepted	Correct decision of probability 1-α	Wrong decision of probability β, ***type II error***
Hypothesis H_1 is accepted	Wrong decision of probability α, ***the type I error***	Correct decision of probability 1- β, **power (sensitivity)**

Probability to make *the type I error* should be small, because must be *the weighty arguments* for recognition (for example) that one method of treatment is better than another. This **probability** p is known as a **significance level α**.

The significance level α is called probability of rejection of a null hypothesis when it is in fact true.

Fig. 1

More serious consequences of the type I error, it is necessary to take lesser a significance level. In medical examinations usually is used $\alpha = 0.05$, or $\alpha = 0.01$ and value of $\beta = 0.2$ or 0.1. It is desirable, that α and β must be as small as possible that can be reached only by increasing a sample size. This conclusion is very important because it is directly connected with the plan of experiment.

Reasonable **parity between α and β** find, proceeding from weight of consequences (damage) of each error. For example, testing hypothesis H_0 about absence at the patient of the certain disease, and an attribute of

disease is value of arterial pressure (AP). Then H_0: AP in norm, i.e. the patient is healthy; H_1: AP differs from norm, i.e. the patient is sick. Then the type I error (rejection H_0, when it is true): patient is sick, when he is in fact healthy. Consequences of this mistake are inconveniences for the patient who, for example, should pass additional diagnostic or treatment. Other situation in case of the type II error: to recognize the person able-bodied, when he really is sick. Actually there is a refusal from treatment of the patient, it is wasted time and consequences of the type II error can be great for the patient. So, in our example with the purpose to reduce probability of the type II error necessary to not take the high level of significance α (for example, to accept α equal to 0.05, instead of 0.01). Opposite situations are possible also. Values of $\alpha < 0.01$ are used at statistical detection of toxiferous medical preparations, when the major meaning has the guarantee from wrong rejection of the tested hypothesis.

So, at selecting of hypotheses *a null hypothesis* (in comparison with the alternative) should be hypothesis, *which is **more dangerously** to falsely reject.*

Statistical test. Critical regions ("tails").

For testing of a hypothesis is used a random quantity K that is function on the sample data and that is called statistical test.

***The statistical test** is the rule (formula) permitting according to sample to accept or reject a null hypothesis.*

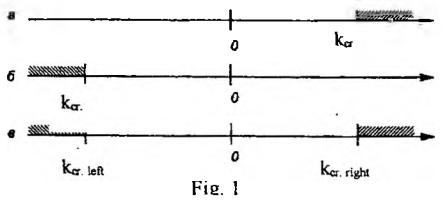
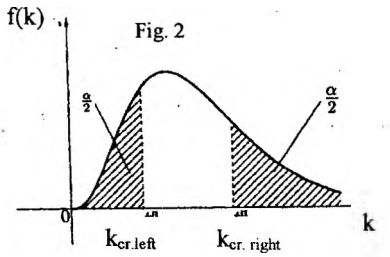


Fig. 1

The statistical test being a random quantity has any probability distribution, for example: normal distribution, Student's distribution, Fisher distribution, $\chi 2$-distribution etc.

Depending on the accepted significance level α from all area of allowable values of test K is allocated (see fig. 1) *critical* region. It makes with help of number $\kappa_{cr.}$, that find with help of tables of distribution of each test K. Further works the following rule (**basic principle of testing of statistical hypotheses**): *if the observed value k_{obs} of the test K calculated on sample gets to the critical region, zero hypothesis H_0 is rejected for the benefit of*



alternative H_1 and if $k_{obs.}$ not gets, H_0 is accepted.

Critical region depending from selection of κ_{cr} can be *unilateral* (right-tailed or left-tailed) or *two-tailed* (fig. 2, 3, 4). For right-tailed (left-tailed) critical region, value K satisfy to the condition: $P(K \geq \kappa_{cr.})=\alpha$ and $P(K \leq \kappa_{cr.})=\alpha$, where P (...) = α is probability, that test K has the value greater (or accordingly smaller) than κ_{cr}. P (...) = α is equal to the area of the right or left "tail" on the graph of distribution of probabilities (fig. 3). Similarly, for two-tailed critical region $P(K \leq \kappa_{cr})+ P(K \geq \kappa_{cr})=\alpha$, i.e. value α is the area of both "tails" on the graph of distribution of probabilities (fig. 2).

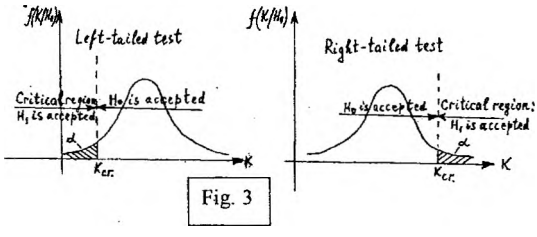


Fig. 3

The unilateral critical region should be used, when process interesting us should go only in one direction. For example, there are weighty arguments to assert that the certain diet will necessarily decrease weight of a patient. But even in this case it is necessary to make secure having chosen two-tailed critical region. In our example it means, that at some people the offered diet can result in increase in weight. Therefore, as practice shows, in the majority of researches the *two-tailed test is applied.*

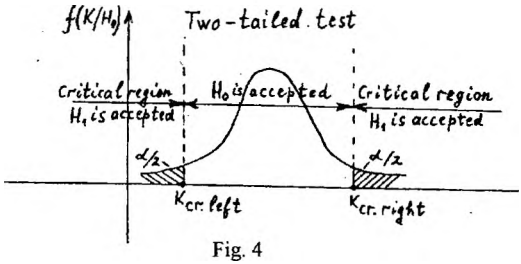


Fig. 4

There are parametrical and nonparametric tests. **Parametrical tests** are used if samples are taken from population with the known law of distribution, for example, the normal law of distribution. Normality of distribution of sample should be statistically proved before application of parametrical test.

Nonparametric tests are used, if there is no submission of distribution of sample to the normal law. For example, if the volume of sample is so small, that it is impossible to estimate the law of distribution of the data in sample. Parametrical tests are more powerful, than nonparametric in detection of real effect.

Procedure of testing of hypotheses

A typical procedure consists of the following steps.

1. From one or several populations is collected the initial statistical material as two or some samples.
2. A researcher formulates the basic (H_0) and alternative (H_1) hypotheses, and also chooses a significance level α (0.01 or 0.05) corresponding to the purposes of researches.
3. Must be selected the test **K**, which approaches in the given situation and is defined, what test is necessary (right-tailed, left-tailed or two-tailed) and then under corresponding formulas we calculate value of statistical test κ_{obs} **(sample statistic)** for given data (samples).
4. Under the tables, corresponding to the chosen method we find the critical value $\kappa_{cr.}$ for the accepted significance level (fig. 3 and fig. 4).
5. It is made a conclusion on validity of the hypothesis Ho or H_1:

If values of the sample statistic $\kappa_{obs.}$ are in the critical region, the basic hypothesis H_0 is rejected and accepted alternative hypothesis H_1(distinctions between observable values and theoretical are significant, i.e. are caused by error of zero hypothesis). **If values of the sample statistic do not get to the critical region, the hypothesis H_0 is accepted** (distinctions are not significant and caused by the casual reasons) (see fig. 3 and 4).

What is p <0.05?

During of testing of statistical hypotheses except of calculation of statistical test K, in the modern statistical packages is calculated corresponding value p, where p is a probability that the data corresponds to null hypothesis H_0. Small values of p testify about "wonderfulness" of such event and results, that H_0 is rejected. Usually H_0 is rejected, when p-values are less *than* 0.05.

Comparing received value p with the accepted significance level α, we make a conclusion about hypotheses:

if $p > \alpha$ (α is the accepted significance level, usually is 0.05), **then H_0 is accepted (distinctions are non significant);**

if $p < \alpha$, H_0 is rejected (distinctions are statistically significant at p <0.05).

Using of round numbers 0.05; 0.01, etc. as a significance level is a consequence of manual statistical calculations in precomputer time. Now it is recommended to specify exact value of p (to within three marks), that allows the reader to estimate independently the statistical importance of result, for example, values p=0.049 or p=0.051 should be interpreted practically equally.

Dependent and independent samples

Two samples are **dependent** if the values in one are related to the values in the other in some way. Two samples are **independent** if the values in one are not related to the values in the other. Example of *independent* samples: parameters (for example, arterial pressure) of two groups of patients to which were applied different techniques of treatment.

Examples of *dependent* (connected samples):

1) parameters *of one* group of patients *before and after* influence of any factor, for example,

techniques of treatment;
2) parameters of the different parts of the same object, for example, a condition of two finitenesses, one of which was exposed to treatment,and the second is not.

Let's pass to consideration of some most popular statistical hypotheses used in medical researches and examples of their using.

Test comparing two means: Student's test

The similar problem arises at comparison of two samples, for example, two groups of the patients, undergone to particular action (for example, treatment on some procedures of two groups of patients, one of which accepts a particular medicinal preparation, and other, control group, accepts placebo(the medicinal form containing neutral matters). Thus comparison of two means allows to judge about degree of action, about significance of a possible effects or its absence.

1. Target setting. From two populations *X* and *Y*, distributed under the normal law (testing of normality of both samples will necessarily be spent beforehand), with *equal variances are obtained two* samples of volumes n_X and n_Y accordingly. It is required to compare means of the relevant populations.

Let's consider order of the steps at solution of this problem according to the order set above.

2. Tested hypotheses:

H0: $M(Y_p)=M(X_p)$ (means of two populations are equal);

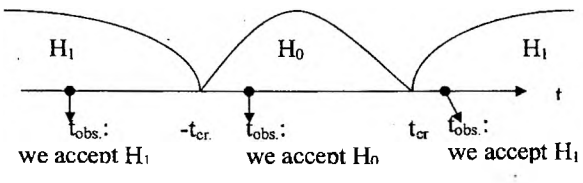
H1: $M(Y_p)\neq M(X_p)$, i.e. we take two-tailed critical region.

3. For a testing of hypothesis about equality of means of two populations at equal variances is applied Student's test, which sample statistic value $t_{obs.}$ is calculated under the formula:

$$t_{obs.} = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{(n_X-1)\cdot S_X^2+(n_Y-1)\cdot S_Y^2}{n_X+n_Y-2}}}\cdot\sqrt{\frac{n_X\cdot n_Y}{n_X+n_Y}},$$

where S_λ^2 and S_Y^2 are sample variances, $\bar{x}$ and $\bar{y}$ are means of samples.

Fig. 5



4. From the table of t-distribution we take for a confidence level α =0.05 two-tailed critical region. For this purpose beforehand we determine number of degrees of freedom under the formula:

$$f=n_X+n_Y-2$$

Critical region for rejection of H_0 (fig. 5):

$$|t_{obs.}| > t_{cr}$$

Test comparing two variances: Fisher F-test.

In many clinical examinations is important to test the hypothesis about equality of two populations variances of two *normal* samples. This problem can be solved with help *of F-test of Fisher.* The similar problem of comparison of variances arises in case of comparison of precision of measurements, precision of devices and comparison of methods. As the variance characterizes degree of dispersion of values concerning of mean, then the best method has minimal variance.

1. *Target setting.* For random quantities X and Y, distributed under the normal law, are obtained two samples of volumes n_X and n_Y accordingly. It is required to compare variances of the relevant populations. Let $\alpha = 0.05$ or 0.01.

2. Tested hypotheses: $H_0 : D_p(Y) = D_p(X)$ (populations variances are identical).

Alternating hypothesis $H_1: D_p(Y) \neq D_p(X)$ (two tailed critical region).

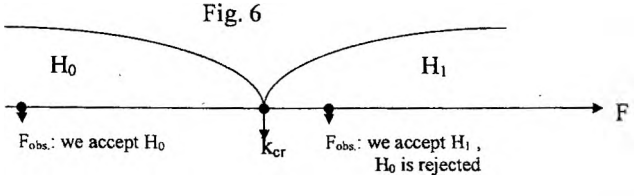
3. For a testing of hypothesis about equality of variances we will use F-test. We compute concrete values of sample variances S_X^2 and S_Y^2, also we find the ratio (observed value or sample statistic of test):

$$F_{obs.} = \frac{S_L^2}{S_S^2},$$

where S_L^2 and S_S^2 are larger and smaller of numbers S_X^2 and S_Y^2.

4. Further under the table of F-distribution for the given confidence level α and for numbers of degrees of freedom $\kappa_1 = n_L - 1$ and $\kappa_2 = n_S - 1$ we find a critical value $\kappa_{cr} = F_{cr}(\frac{\alpha}{2}; \kappa_1; \kappa_2)$. It is proved, that in this case the two-tailed critical region can be exchanged by right-tailed, that is if $F_{obs} < \kappa_{cr}$ hypothesis H_0 is accepted, if $F_{obs} > \kappa_{cr}$ difference of variances is significant and H_0 is rejected (fig. 6).

Fig. 6



Nonparametric tests: Wilcoxon-Mann-Whitney test (U-test).

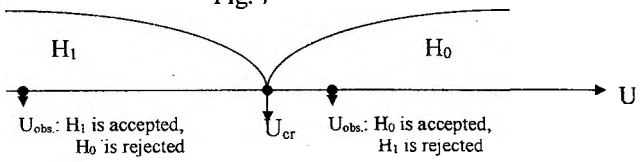
The given test is *nonparametric analog of the t-Student test* and is used for testing of a hypothesis: two *independent* samples belong to the same population. Here there is no necessity of the sample normal distribution. The

nonparametric tests are based on ranks that are numbers 1, 2, 3 ..., featuring their position in a ranked data set.

1. *Target setting*. For random quantities X and Y with unknown laws of distribution obtain samples of volumes n_X and n_Y. Values of elements are submitted in a serial scale. It is tested a hypothesis: compared independent samples belong to the same population. α=0.05 or 0.01.
2. Tested hypotheses: H_0: M(X)=M(Y). H_1: M(X)≠M(Y).
3. For calculation of U-test it is necessary:

- to arrange numerical values of samples to one general row;
- to enumerate terms of general row from 1 up to $N=n_1+n_2$, where n_1 and n_2 are volumes of the first and second samples. These numbers will be ranks of terms in the row. If there are identical values of sample elements, then for these elements give the identical ranks, equal to arithmetic mean of ranks of identical elements;
- for each sample find the sum of ranks R_1 and R_2;
- find values U_1 and U_2 (observed values of test):

Fig. 7



$U_1 = R_1 - \frac{n_1(n_1+1)}{2}$ and $U_2 = R_2 - \frac{n_2(n_2+1)}{2}$,

and to pick U_{obs} as smaller from U_1 and U_2;

4. Under the table of critical values of U-test at the given confidence level find U_{cr}. If $U_{obs} > U_{cr}$ H_0 is accepted (differences statistically non significant, see fig. 7).

Execution of the work

Solve the next problems:

1. Sample data were collected in a study of calcium supplements and the effects on blood pressure. A placebo group and a calcium group began the study with measures of blood pressure (mmHg):

Placebo (X): 124.6; 104.8; 96.5; 116.3; 106.1; 128.8; 107.2; 123.1; 118.1; 108.5; 120.4; 122.5; 113.6.

Calcium (Y): 129.1; 123.4; 102.7; 118.1; 114.7; 120.9; 104.4; 116.3; 109.6; 127.7; 108.0; 124.3; 106.6; 121.4; 113.2.

At the significance level α =0.05 test:

a) the equality of dispersions of two groups of patients by Fisher F- test;
b) the hypothesis H_0: M(Y)=M(X) at alternative H_1: M(Y)≠ M(X) by means of t-test of Student.

2. Sample data were collected in a study to determine whether a drug affects eye movements. The drug is given to one group, while a control group is given a placebo that produces no effects. Test the claim (by the Wilcoxon-Mann-Whitney-test) that the drug has no effects on eye movements ($\alpha = 0.05$):

Drugged group: 652, 512, 711, 621, 508, 603, 787, 747, 516, 624, 627, 777, 729.
Control group: 674, 676, 821, 830, 565, 821, 837, 652, 549, 668, 772, 563, 703, 789, 800, 711, 598.

Control questions

1) What is a statistical hypothesis? Give some examples.
2) What is a confidence level? What confidence level does frequently use in medicine?
3) What are type I error and type II error?
4) What is the statistical test? Why in many researches is used two-tailed test?
5) Formulate the basic principle of testing of statistical hypotheses.
6) What is the difference between parametrical and nonparametric tests?
7) Procedure of testing hypotheses.
8) What means $p < 0.05$?
9) Give examples of dependent and independent samples.

Laboratory work №3

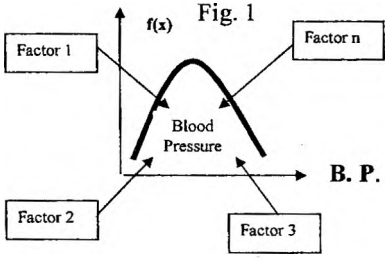
Distribution fitting: testing of sample for normality

Purpose of the work: to master the basic modes of examination of sample on normality.
Equipment: statistical tables; computer program Statistica 6.

Theory of the work

One of the first problems at statistical analysis of the medico biological data is checkout of correspondence of data to the normal distribution law. The symmetrical bell-shaped curve of the normal law has been opened by A. Moivre in 1733 and explored by K. Gauss (1809), and the term "normal distribution" belongs to K. Pearson. The majority of statistical procedures in the modern packages (t-criterion of Student, analysis of variances, correlation analysis on Pearson, etc.) are oriented on the samples received from population with normal distribution. But only about 20 % of the distributions meeting in medico biological examinations are approximately normal. Therefore use of the above-stated parametric procedures for the data which distribution does not submit to the normal law can lead to untrue conclusions.

Fig. 1



What are the requirements the data can be distributed accordingly to normal law? The answer to this problem is given ***by the central limit theorem in the form of Lyapunov's theorem.*** Sense of the theorem: the sum of n independent random variables set by the arbitrary distributions at increase of n has distribution which is tended to normal, provided that influence of each random quantity is insignificant in comparison with their net effect. The considerable number of random phenomenon in the nature, technics, medicine flows past under such plan. For example, arterial pressure (Blood Pressure) depends on many factors: mode of life (the factor 1), heredity (the factor 2), weather requirements (the factor 3) etc., and influence of many of them can be inappreciable. According to the **central limit theorem**, always, when it is possible to suppose that considered quantity (for example, Blood Pressure) is defined by the sum of the big number of random factors, influence of each of them is insignificant, then its distribution will be close to the normal. For a concrete random quantity it means *its correlation with many subsystems of human organism*, instead of with one-two of them. The same normal distribution have the

next random quantities: distribution of human height, mass, level of hormones, outcomes of the random experiment depending on many small factors, recording errors in metres, etc. Therefore the understanding of the nature of the defined data facilitates checkout testing on normality.

It is possible to select some stages of testing of the data on normality.

1). **Qualitative testing** is a plotting of **histogram** of distribution of a random variable with which help is possible to evaluate visually how much the histogram is close to the normal distribution graph. As the histogram form is very sensitive to the number of intervals into which the range of random variables is divided, it is recommended to determine the number of intervals by formula of Sterdgess:

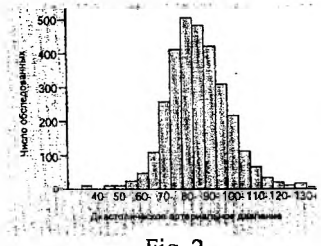


Fig. 2

$k = 1 + 3.33 \lg n$. Besides, it is necessary to consider that for any symmetrical sample the histogram will have any shift to the left or to the right that is theoretically justified (for example, owing to a procedure of takeoff of the data).

2). The second **visual** mode of testing on normality is build-up **of probits-graphs**: on axis X put aside observable value, and on an axis Y – expected value. On a degree of deviation of the probit-graphs from a straight line we can estimate affinity of distribution to normal and also about presence of outliers. On fig. 3 is shown the example of distribution close to normal and on fig. 4 is shown the example of the distribution that is distinct from the normal.

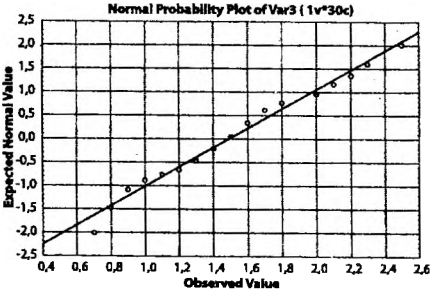


Fig.3

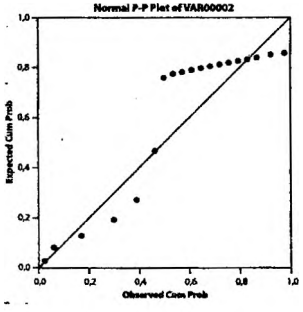


Fig. 4

3). Visual methods on the basis of histograms and the probit-graphs can give only qualitative initial assumptions about kind of distribution. Conclusions on the basis of **numerical characteristics of sample** are more reliable. At first, having computed beforehand mean $\bar{x}$ and standard deviation S of the given sample is possible to muster **«rule of three sigma»**, according to which it is almost authentic (p=99.7 %) that normally distributed random variable gets to the interval $\pm 3S$ in the individual trial.

At a great number of observations (n› 100) good results gives by calculation of the parameters of the shape of distribution: **coefficient of skewness, kurtosis and their standard errors.** Coefficient of skewness As (Skewness) is a performance bias of distribution to the right or to the left concerning maxima. For symmetrical distributions the coefficient of skewness is equal to null: As =0. If As‹ 0 the distribution curve is skewed to the left concerning symmetrical curve (the left tail is more long right), and if As› 0 the curve is skewed to the right (the right tail of the frequency histogram is more long left).

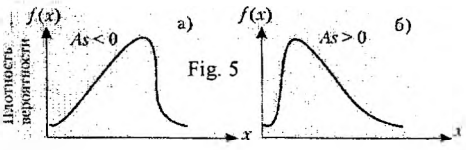


Fig. 5

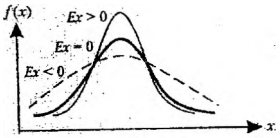


Fig. 6

Distribution is considered symmetrical, if |As | ≤0.1 and dissymmetric, if |As|› 0.5. The coefficient of kurtosis Ex (Kurtosis) is a performance of flat-toppedness of distribution curve. For a normal curve the coefficient of kurtosis is equal to null: Ex=0. Distribution is close to normal, if |Ex |≤ 0.1 and considerably deviates it, if |Ex|› 0.5. Thus discovered **standard errors of As and Ex should have the same order as value of As and Ex.**

The coefficient of skewness, the coefficient of kurtosis and standard error of kurtosis are computed under the formulas:

$$As = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^3}{n \cdot s^3}, \quad Ex = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^4}{n \cdot s^4} - 3, \quad m_{Ex} = 2 \cdot m_{As} = 2\sqrt{\frac{6}{n+3}}.$$

The box graph of Tuki (Box-and-whisker diagram) gets summarised information about distribution into small volume (some variants see on fig. 7). The box shows the interquartile (quartiles are the values dividing sample on 4 equal parts) interval, the horizontal line in the box shows median line, and a sprocket wheel shows the mean. "Whiskers" shows a range of distribution, for example: in limits from 5 to

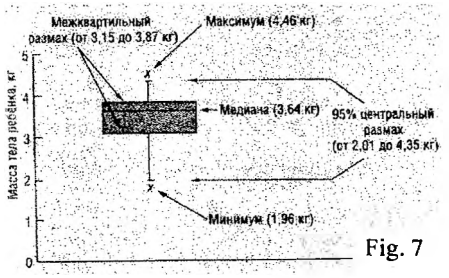


Fig. 7

95 percentiles (percentiles are the numbers dividing sample on 100 equal parts), by daggers (or points) are shown the minimal and maximum magnitudes. If the data are distributed normally (the middle chart) a whiskers will have equal length, the median line bisects a rectangle and will coincide with the mean.

4). The most convincing results at normality testing are given by use **of goodness-of-fit tests** – the statistical tests designed for testing of the consent of empirical data and theoretical model. Null hypothesis H_0 is set: sample is obtained from a normal population and alternate hypothesis H_1: the population distribution differs from the normal. The significance level corresponding to received value of statistics of test further calculates: if $p > 0.05$, a null hypothesis about normality of sample is accepted; if $p < 0.05$ H_0 rejects and accordingly accepts H_1.

Most often are used following statistical tests for testing normality:

1) χ^2 for discrete and continuous variables; $n > 30$.
2) W – test of Shapiro-Wilk for small samples: $3 < n < 50$; mean value and the standard deviation of sample are in advance unknown and are computed on sample;
3) Kolmogorov-Smirnov' test (or its modification of Lilliefors) is for continuous (or sometimes discrete) variables; it is more power, than χ^2. Mean value and standard deviation of sample are known in advance; $n > 50$.

In medico-biological examinations are often used small samples, in this case for normality testing is recommended to use the Shapiro-Wilk's test designed for samples with number from 3 to 50 observations. If $n \leq 10$, irrespectively of outcomes of testing on normality, it is recommended to apply nonparametric methods. The given test is preferable, since is the most strength and universal and thus is most strict of enumerated above.

If the hypothesis about a normal distribution is rejected, but there are falling out value after their excision it is necessary to carry out the test for normality once again.

Let's consider an example of testing of distribution on normality by means of test of Shapiro – Wilk (by means of calculation).

Example. At 6 students have measured systolic pressure. Test the data on normality.

1	2	3	4	5	6
110	115	160	130	120	130

1. We set hypotheses: H_0: the data is obtained from a population with the normal distribution; H_1: distribution differs from the normal. The significance level is $\alpha = 0.05$.
2. We calculate values of the sample mean $\bar{x}$ and the sample variance s^2:

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_6}{6} = \frac{110 + 115 + \ldots + 130}{6} = 127.5.$$

$$s^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2 = \frac{1}{6-1}\left[(110 - 127.5)^2 + \ldots + (120 - 127.5)^2\right] = 317.5.$$

3. We arrange sample values in ascending order (we range sample) as it is shown in the 2 column of the table.
4. We organize differences Δ_k, for this purpose from maximum value x_n is subtracted the least x_1, then from x_{n-1} is subtracted x_2 and etc. If n is an even number (as in our case) then number of differences is $k = n/2$, and if n is an

odd number then $k = \frac{n-1}{2}$, thus central variant in formation of differences does not participate.

1	2	3	4	5	6
№	x_i	k	Δ_k	a_{nk}	$a_{nk} \cdot \Delta_k$
1	110	1	50	0.6431	32.155
2	115	2	15	0.2806	4.209
3	120	3	10	0.0875	0.0875
4	130				$\Sigma a_{nk} \cdot \Delta_k$=36/4515=b
5	130				
6	160				

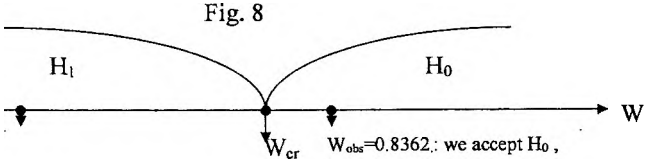
Numbers of differences k are placed in the column 3 and values of differences Δ_k are placed in the column 4.

5. Under application tables we find values of coefficients a_{nk} of W- test of Shapiro-Wilk for n = 6 and corresponding k. This value is placed in the 5 column of the table.
6. We calculate products $a_{nk} \cdot \Delta_k$ and put it in the column 6.
7. We compute $\Sigma a_{nk} \cdot \Delta_k$=36.4515=b.
8. We compute observed value $W_{obs.}$(test statistic) of the test by formula:

$$W_{obs.} = \frac{b^2}{(n-1) \cdot s^2} = \frac{36.4515^2}{(6-1) \cdot 317.5} = 0.8369.$$

9. From the application table we find critical value $W_{cr.}$ of W - test of Shapiro-Wilk: $W_{\alpha, n} = W_{0.05, 6} = 0.762 = W_{cr.}$
10. W - test of Shapiro-Wilk constructs so that hypothesis H_0 accepts, if $W_{cr.} < W_{obs.}$ (unlike remaining tests for which H_0 accepts, if $W_{obs.} < W_{cr.}$), that is observed at us, i.e. 0.762 <0.8369.

Fig. 8



Conclusion: *we accept the null hypothesis that is the data is taken from a population distributed under the normal law.*

Let's consider now how to apply the test of Shapiro-Wilk by means of package Statistica 6.0. After input of datas we sequentially take following commands: Statistics; Basic Statistics/Tables; Descriptive Statistics; Normality (Y Shapiro-Wilk's W - test).

The result of evaluations is shown on fig. 9, whence it is visible that observable value of the test is $W_{obs.} = 0.87367$ (compare to earlier computed variant $W_{obs.} = 0.8369$), the significance level $p=0.24123 > 0.05$, and means the null hypothesis about normality of distribution accepts.
From the same figure is visible that the visual approach grounded on **build-up of a histogram** is suitable only for approximate testing on normality.

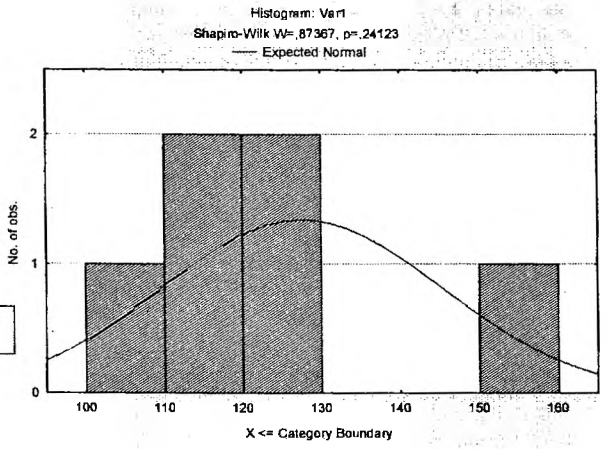


Fig. 9

More informative visual approach is plotting **of probits-graphs**: Statistics; Graphs; 2D Graphs; Normal Probability Plot. The result for our example is shown on fig. 10.

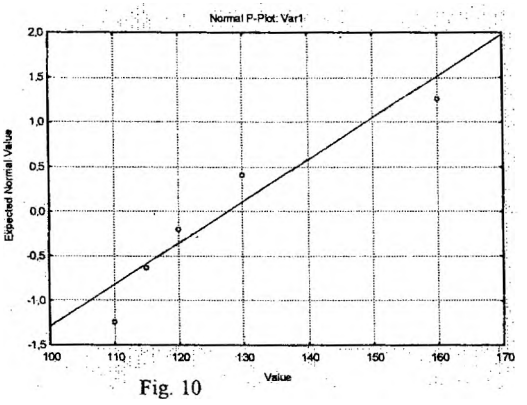


Fig. 10

If is observed about the symmetrical deviation of points (arc-wise or in the form of letter S), as on fig. 10, then distribution is close to the normal.

Let's compute the values **of the coefficient of skewness, the coefficient of kurtosis and the kurtosis standard error** (Standard error of Kurtosis) under formulas:

$$As = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^3}{n \cdot s^3} = 0.786, \qquad m_{As} = \sqrt{\frac{6}{n+3}} = 0.816,$$

$$Ex = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^4}{n \cdot s^4} - 3 = -0.954, \quad m_{Ex} = 2 \cdot m_{As} = 2\sqrt{\frac{6}{n+3}} = 1.6329.$$

As values of the coefficient of skewness and standard error of skewness on the one hand, the coefficient of kurtosis and the kurtosis standard error on the other hand have *the same order* is possible to draw a conclusion on affinity of allocation to the normal also. Such approach also is only approximate. For computer determination of the above-stated coefficients we can use following steps: Statistics; Basic Statistics/Tables; Descriptive Statistics; (Y Skewness; Y St. Err. Skewness; Y Kurtosis; Y St. Err. Kurtosis).

Work execution

1. Testing of normality of distribution by means of Shapiro-Wilk's test.

At 6 students of bunch have measured systolic pressure. Test the data on normality (without using of statistical packages).

1	2	3	4	5	6
120	120	110	130	110	150

2. Using p. Statistica, test the sample on normality with the help of: plotting of histogram; plotting of probits-graphs; calculation of parametres of the shape of distribution (including the rule of "three sigmas") by means of Shapiro-Wilk's test.

Control questions

1. What is the normal distribution? Give its properties, give any examples.
2. What are requirements according to Lyapunov's theorem that any distribution can be referred to the normal?
3. Describe qualitative and visual ways of testing on normality. What are imperfections of these ways?
4. What are the characteristics of the shape of distribution? When they characterize a normal distribution?
5. What are the goodness-of-fit tests, formulate requirements and the order of its application.
6. What are the advantages of Shapiro-Wilk's test? What is the requirement to accept the null hypothesis?

Laboratory work №4

Testing of statistical hypotheses: chi square test

Purpose of the work: to familiarize with the most used nonparametric statistical test.

Equipment: tables of corresponding criteria.

Theory of the work

Chi square distribution.

The term "chi square" refers both to a statistical distribution and to a hypothesis testing procedure that produces a statistic that is approximately distributed as the chi square distribution. At first let's talk about the first sense of this term. We use the Greek symbol χ^2 (pronounced "*ki square*" to rhyme with "sky square") to represent values of the chi square distribution.

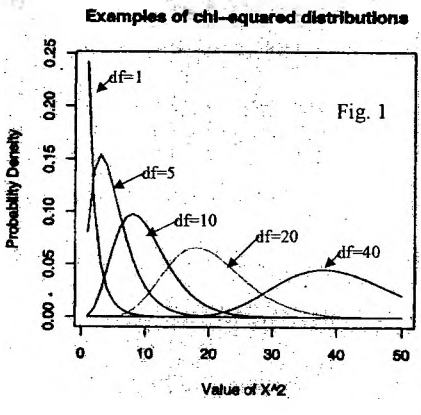


Fig. 1

χ^2-distribution is distribution of probabilities of sum of squares of n independent variables X_1, X_2,..X_n that have standard normal distribution with mean of 0 and standard deviation of 1:

$$\chi^2 = X_1^2 + X_2^2 + ... + X_n^2.$$

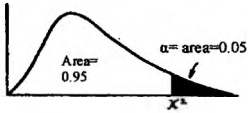
Properties of the χ^2 distribution (see fig 1):

1. It is not symmetric.
2. The values of χ^2 are positive (i. e. $\chi^2 > 0$).
3. It is asymptotic to the horizontal axis on the right -hand-side.
4. The shape of the χ^2-distribution depends upon degrees of freedom *df*, just like Student's t-distribution.
5. If *df* increases, the χ^2-distribution becomes more symmetric.
6. Total area under the curve is equal to 1. The P-value is the area to the right under the density curve.

Critical values of the χ^2-distribution are found in the table 1. Find the critical value of the shi square for a right-tail with α=0.05 and df=15. Answer: on the crossing of the row=15 and column=0.05 we have $\chi^2(0.05;15)$=24.996 (see table 1).

Table 1 The Chi-Square (χ^2) Distribution

Area to the Right of the Critical Value



Degrees of freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15,987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267

Types of data. Contingency tables.

The chi square test is the *most important and most used* member of the nonparametric family of statistical tests. Nonparametric statistical procedures test hypotheses that do not require normal distribution and homogeneity of variances about the populations from which the samples were drawn and are designed for categorical (ordinal and nominal data). Chi Square test is used to investigate whether distributions of categorical variables differ from one another. There are basically two types of random variables and they yield two types of data: numerical and categorical (see fig. 2). Notice that discrete data arise from a counting process, while continuous data arise from a measuring process. Suppose you conducted a drug trial on a group of animals and you tested that the animals receiving the drug would show increased heart rates compared to those that did not receive the drug. Tables similar to tables 2 (general form) and 3 (example) are generally called contingency tables, or two-way tables. The word *contingency* refers to dependence, and the contingency table serves as a useful medium for analyzing the dependence of one variable on another.

Fig. 2

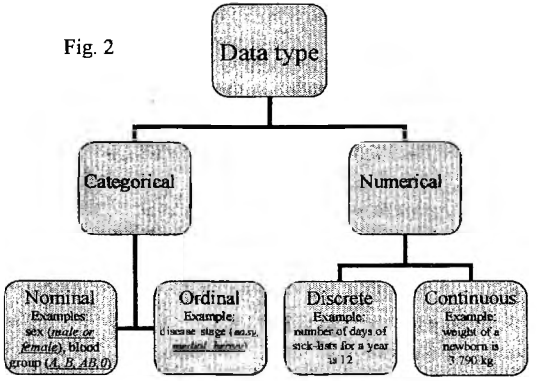


Table 2

Variable 2	Variable 1: Data type 1	Data type 2	Total
Category 1	a	b	a+b
Category 2	c	d	c+d
Total	a+c	b+d	a+b+c+d

Table 3

	Heart rate increased	No heart rate increased	Total
Treated	36	14	50
Not treated	30	25	55
Total	66	39	105

Types and purpose of the chi square.

There are several types of chi square tests:

1. A **test of goodness of fit** establishes whether or not an observed frequency distribution differs from some theoretical expected distribution (normal, for example, see fig. 3,a).

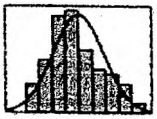
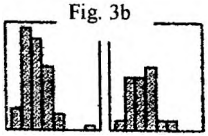


Fig. 3,a

2. A **test of independence** assesses whether paired observations on two variables, expressed in a *contingency table*, are independent of each other. In other words, test of independence is a way of determining whether two categorical variables are associated with one another in the population. The chi square test of independence is a test regarding a sample from a *single* population.

Fig. 3b



3. Test that two or more samples have the **same distribution (test of homogeneity,** see fig. 3,b). For example, do people of different races have the same proportion of smokers to non-smokers. Test of homogeneity can be used to compare the population proportions from *different* populations.
4. Test of presence of the **linear trend.**

In medical practice more often are used 1st and 2nd types of the chi square test. The *procedure* of the test includes the following steps:

1. Determination the degrees of freedom df (number of degrees of freedom corresponds to the number of values that are free to vary):

df = r-1, if there is one independent variable, r is the number of levels of the independent variable;

df = (r-1) (s-1), if there are 2 independent variables, where r and s are the numbers of levels of the first and second variables respectively;

df = (r-1) (s-1) (t-1), if there are 3 independent variables, where r, s and t are the number of levels of 1st, 2nd, and 3d variable respectively.

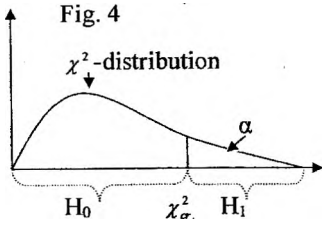
2. Calculation the chi squared test-statistic χ^2:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i},$$

where: χ^2 is Pearson's test statistic (which asymptotically approaches a χ^2-distribution),

O_i is an observed frequency,
E_i is an expected (theoretical) frequency,
n is a number of cells in the table.

Fig. 4



3. **Conclusion:** compare test statistic χ^2_{obs} to the critical value χ^2_{cr} (right tale): if $\chi^2_{obs.} > \chi^2_{cr}$, H_0 is rejected, H_1 is accepted; if $\chi^2_{obs.} < \chi^2_{cr}$ => H_0 is accepted (fig. 4).

When there is only one degree of freedom (table 2x2), the correction known as **Yates correction** can be used:

$$\chi^2_{obs.} = \sum \frac{(|O_i - E_i| - 0.5)^2}{E_i}$$

Explanation degrees of freedom. Number of degrees of freedom corresponds to the number of values that are free to vary. What it means? Degree of freedom characterizes how many of the numbers in our data are determined independently of the rest. For example, if we had 2 numbers that had to add to 10, and one were 4, the other would have to be 6. We could pick anything for the first number, and determine it independently, but the second number would then have to have a particular value in order for the sum to be 10. If we had 3 numbers that had to add to 10, the first two could be independent, but the third would have to have a particular value to add to 10.

Test of goodness of fit

Example 1.

A random sample of 100 people (44 men and 56 women) has been drawn from a population in which men and women are equal in frequency. Is the observed difference statistically significant? (In other words, is in the given population number of men the same as number of women?) The observed value of men and women would be compared to the theoretical frequency of 50 men and 50 women (Uniform distribution).

Test of goodness of fit is an inferential procedure used to determine whether a given frequency distribution follows a claimed distribution. It is a test of the agreement of conformity between observed frequencies O_i and the expected frequencies E_i for several classes of categories.

Assumptions:

-the data are randomly selected,

- all expected frequencies (observations) are ≥1,

-no more than 20% of the expected frequencies are less than 5.

Step 1. A claim is made regarding a distribution.

H_0: the random variable follows the claimed distribution.

H_1: the random variable does not follow the claimed distribution.

Step 2. Select a significance level α, find the number degrees of freedom df=k-1, where k is number of categories. Find the critical value of the chi-square: χ^2_{cr} (α, k-1). Note that test goodness of fit is right-tailed test, so the critical value is always on the right tale (fig. 4).

Step 3. Calculate the test-statistic:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i},$$

where:

O_i is an observed frequency,

E_i is an expected (theoretical) frequency: $E_i=np_i$ for each of the k categories , n is a number of trials, p_i is probability of the i-th category.

Step 4. Draw a conclusion:

a. compare the test statistic with the critical value: if $\chi^2_{obs.} > \chi^2_{cr.}$, H_0 is rejected, H_1 is accepted;

b. interpret the conclusion in the context of the problem.

Example 1. Solution.

Step 1. Null and alternative hypotheses are

H_0: the random variable (number of men and women) follows the claimed distribution (Uniform distribution).

H_1: the random variable does not follow the uniform distribution.

Step 2. Significance level is α=0.05. Number of degrees of freedom is df=2-1=1. Critical value of the chi-square: χ^2_{cr} (α, k-1)= χ^2_{cr} (0.05; 1)=3.841(see the table 1).

Step 3. We put in the contingency table observed and expected frequencies and calculate the test statistic (with **Yates correction**):

	Observed	Expected
Men	44	50
Women	56	50
Total	100	100

$$\chi^2_{obs.} = \sum \frac{(|O_i - E_i| - 0.5)^2}{E_i} = \frac{(|44-50| - 0.5)^2}{50} + \frac{(|56-50| - 0.5)^2}{50} = 1.21.$$

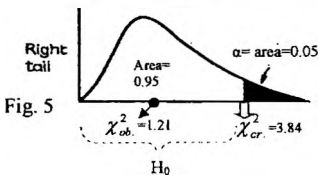


Fig. 5

Step 4. Conclusion: because of $\chi^2_{obs.} < \chi^2_{cr}$ => H_0 is accepted (see fig. 5), i. e. in the given population number of men is the same as the number of women. The observed difference is not statistically significant at 0.05 level.

Test of independence

Example 2.

In a small town was studied gastroenteritis episode. Researchers have guessed that tap water was the source of infection. They researched association between an amount of the drunk water and number of the diseased.
What conclusions can be made of the data resulted in the table?

Amount of the drunk water, number of glassfuls per a day	Number of diseased	Number of not diseased
Less than 1	39	121
From 1 to 4	265	258
5 and more	265	146

In the given contingency table the **row variable** is a number of glassfuls and the **column variable** is a number of diseased.
The chi square test of independence is used to find out whether there is an association between a row variable and column variable in a contingency table constructed from a sample data.
The test statistic is

$$\chi^2 = \sum \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}},$$

where i and j are indexes the rows and columns of the table,
O_{ij} are the observed counts in the cell of the table,
E_{ij} are the expected frequency (counts) in the cell:

$$\text{Expected frequency} = \frac{(row total) \cdot (column total)}{table total}.$$

Degrees of freedom: df=(r-1)(c-1), where r is the number of rows and c is the number of columns in the contingency table, provided that all expected frequencies are ≥ 1, no more than 20% of the expected frequencies are less than 5.

Assumptions:

-the data are randomly selected,
- all expected frequencies (observations) are ≥ 1,
-no more than 20% of the expected frequencies are less than 5.

Step 1. A claim is made regarding the independence (or dependence) of two variables.

H_0: the row variable and column variable are independent.
H_1: the row variable and column variable are dependent.

Step 2. Select a significance level α, find the number degrees of freedom df: df=(r-1)(c-1).

Step 3. Compute the expected frequencies: $E_i = \frac{(row total) \cdot (column total)}{table total}$.

Compute the test statistic: $\chi^2 = \sum\sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$.

Step 4. Draw a conclusion:

- compare the test statistic with the critical value: if $\chi^2_{obs} > \chi^2_{cr}$, H_0 is rejected, H_1 is accepted;
- interpret the conclusion in the context of the problem.

Example 2. Solution.

Step 1. Null and alternative hypotheses are

H_0: number of glassfuls and number of diseased are independent.

H_1: number of glassfuls and number of diseased are dependent.

Step 2. Let the significance level is α=0.05, the number degrees of freedom is df=(r-1)(c-1)=(3-1)(2-1)=2. Critical value of the chi-square: $\chi^2_{cr}(0.05, 2)$=5.991.

Step 3. We calculate the expected frequencies (shown at the table in the round brackets) and then the test-statistic χ^2_{obs}.

Amount of the drunk water, number of glassfuls per a day	Number of diseased	Number of not diseased	Row total
Less than 1	39 (83.2)	121 (76.8)	160
From 1 to 4	265 (272.0)	258 (251.0)	523
5 and more	265 (213.8)	146 (197.2)	411
Column total	569	525	Table total=1094

$$E_{11} = \frac{(160)\cdot(569)}{1094} = 83.217.\ E_{12} = \frac{(160)\cdot(525)}{1094} = 76.782.\ E_{21} = \frac{(523)\cdot(569)}{1094} = 272.017.$$

$$E_{22} = \frac{(523)\cdot(525)}{1094} = 250.982.\ E_{31} = \frac{(411)\cdot(569)}{1094} = 213.765.\ E_{32} = \frac{(411)\cdot(525)}{1094} = 197.234.$$

$$\chi^2_{obs} = \sum\sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \frac{(39-83.2)^2}{83.2} + \frac{(121-76.8)^2}{76.8} + \frac{(265-272.0)^2}{272.0} + \frac{(258-251.0)^2}{251.0} +$$

$$\frac{(265-213.8)^2}{213.8} + \frac{(146-197.2)^2}{197.2} = 74.8.$$

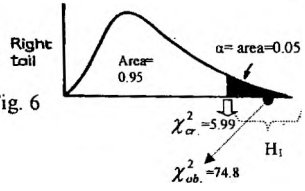


Fig. 6

Step 4. Conclusion: because of $\chi^2_{obs.} > \chi^2_{cr}$ => H_1 is accepted (fig. 6), i. e. number of glassfuls and number of diseased are dependent. A significant relation exists between amount of the drunk water and number of the diseased.

Execution of the work

Solve the next problems, use the chi square test:

1. A die is tossed 120 times. Test the hypothesis that the die is "fair" (see the table).

No. on face of the die	1	2	3	4	5	6
Observ. Fr.	13	28	16	10	32	21

2. Test goodness of fit is often used in genetics to compare the results of a cross with the theoretical distribution based on genetic theory. In a monohybrid cross between two heterozygotes we would have predicted 3:1 ratio of phenotypes. In other words, we would have expected to get 75 A-type and 25 a-type. The phenotypic ratio 85 of A-type and 15 of the a-type are observed. Are the observed and theoretical values have the same distribution? The results are shown in the table:

	Observed	Expected
A-type	85	75
a-type	15	25
Total	100	100

3. Suppose you conducted a drug trial on a group of animals and you tested that the animals receiving the drug would show increased heart rates compared to those that did not receive the drug.

	Heart rate increased	No heart rate increased
Treated	36	14
Not treated	30	25

Is the drug treatment independent with number of animals whose heart rate increased?

4. Many people believe that smoking is unhealthy. In a study of 1000 deaths of males aged 45 to 64, the causes of death are listed along with their smoking habits (see the table). Is the cause of death independent of smoking?

	Cause of death		
	Cancer	Heart disease	Other
Smoker	135	310	205
Nonsmoker	55	155	140

5. There is data of incidence of three types of malaria in the three tropical regions (see the table). Is there association between type of malaria and location?

	Asia	Africa	South America
Malaria A	31	14	45
Malaria B	2	5	53
Malaria C	53	45	2

Control questions

1. Properties of the χ^2-distribution. Types of data. Contingency tables.
2. Explain the purpose of the chi square as a nonparametric statistic.
3. Write the basic steps and computational equation for chi square.

LABORATORY WORK №5

Use of elements of the correlation analysis and a method of the least squares at processing of the medical and biological information

Purpose of the work: 1. To study the method of least squares for a case of linear dependence between investigated quantities.

2. To determine presence and strength of linear correlation of two researched quantities; to find the equation of regress and to find with its help value of dependent quantity.

Equipment: statistical tables.

Theory of the work

Concept of correlation relationship.

Quantities X and Y can be connected by functional and statistical relationships. At **functional** "rigid" relationship between investigated quantities to each value X there corresponds certain value Y, for example, in the law of Ohm I=U/R at R=const to each value U corresponds any value I. Functional relationships are characteristic for laws of physics, chemistry and other natural sciences.

In medical and biologic researches there are **statistical** relationships between quantities more often. For example, at precisely certain change of age of the patient is not observed strictly certain change of arterial pressure (AP). It speaks that change of the AP depends not only on age of the patient, but also from lines of other factors: from sex, a state of health, etc.

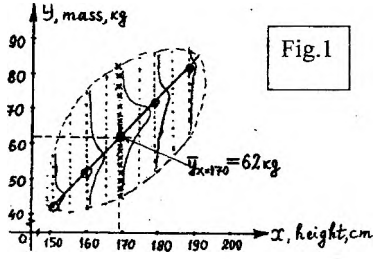


Fig.1

Statistical relationship is caused by the several reasons: 1) influence on Y not only by X, but also other latent factors; 2) inevitability of mistakes at measurement X and Y; 3) biological variability.

Special case of statistical relationship between X and Y is **correlation relationship, when to each value X the population mean (average arithmetic value) of distribution of other value Y is put in conformity.** For example, relationship between a doze of medical product X and its contents in blood Y. On Y influences the weight of the patient, speed of removing of a preparation and other factors, but at the same patient with increase of doze of a medical product its contents in blood unequivocally increases. Other example of correlation relationship is dependence between height of the person and weight.

The person with height for example 170 cm can have weight both 50 kg, and 90 kg, but majority of people of this height have weight in the interval of 60-80 kg , that is to the given height corresponds the distribution of weights close to normal with average value of M (Y). In figure 1 all possible values of weight of the person at *the given* height 170 cm are marked by the dagger, and average value (62kg) is

led round by circle. Clearly, that with increase in height average value of weight M(Y) of a person will grows also, that is we deal with correlation dependence between height X and weight Y:

$$M(Y_x)=f(x). \qquad (1)$$

All set of values X and Y (points on the graph) forms **the scatter diagram** which has been led round by a dotted line in figure 1.

The equation (1) is called **the equation of regress of Y on X**, and its graph is named **a line of regress**. It is similarly possible to describe and inverse correlation dependence $M_{(Xy)} = \varphi(y)$ if it exists. If functions f (x) and φ (y) are linear functions, that it is possible to estimate on character of an arrangement of points of a scatter diagram, then these functions can be presented as:

$$M(Y_x)=ax + b = (\text{Slope})\cdot(x)+(\text{y-intercept}),$$
$$M(X_y)=cy + d$$

For finding of coefficients **a** (Slope) and **b** (y-intercept), included in the equation of a straight line, we use the method **of the least squares.**

Method of the least squares.

In 1806 the French mathematician Lezhandr has proved, that in the best way relationship between X and Y will be reflected by a direct line $\bar{y}_x = ax+b$ for which the condition (see fig. 2) satisfies:

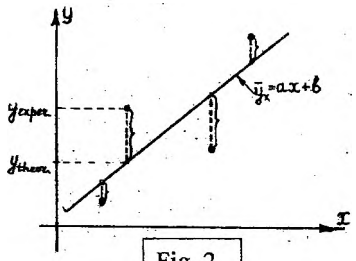


Fig. 2

$$S=\sum_{i=1}^{n}(y_{i\,exp} - y_{i\,theor})^2 = \min, \quad (2)$$

where $y_{i\,exp}$ is value Y received from experience, $y_{i\,theor}$ is calculated y, laying on the straight line, S is a deviation. As $y_{i\,theor}$= =ax+b, condition (2) can be written down

$$S(a,b)=\sum_{i=1}^{n}(y_i - ax_i - b)^2 = \min \qquad (3)$$

*Expression (3) means, that values of coefficients **a** and **b** should be picked up so that the sum of squares of deviations of ordinates of experimental points from ordinates of points of a smoothing straight line would be **minimal*** (fig. 2).

We shall find *a* and *b* without proof and we shall receive the required equation of a straight line $\bar{y}_x = ax+b$:

$$a=\frac{n\sum_{i=1}^{n}x_iy_i - \sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}x_i^2 - (\sum_{i=1}^{n}x_i)^2} \quad (4), \qquad b=\frac{\sum_{i=1}^{n}x_i^2\sum_{i=1}^{n}y_i - \sum_{i=1}^{n}x_i\sum_{i=1}^{n}x_iy_i}{n\sum_{i=1}^{n}x_i^2 - (\sum_{i=1}^{n}x_i)^2} \quad (5).$$

Example 1. Concentration of alcohol (Y_i=c) is measured in blood at n=5 volunteers of identical weight after several portions of alcohol (X_i). By method of the least squares determine coefficients a and b of a smoothing straight line $\bar{y}_x = ax+b$. Construct the graph.

Number of portions, X_i	2	2	4	5	8
Concentration, Y_i	0.05	0.06	0.11	0.13	0.22

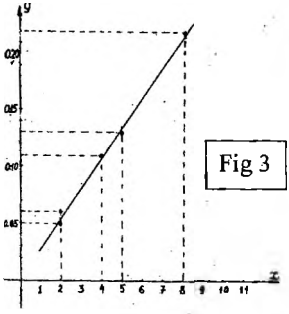


Fig 3

According to (4) and (5) we shall find preliminary $\sum_{i=1}^{n} x_i$, $\sum_{i=1}^{n} y_i$, $\sum_{i=1}^{n} x_i^2$, $\sum_{i=1}^{n} x_i y_i$:

x_i	y_i	x_i^2	$x_i \cdot y_i$
2	0.05	4	0.10
2	0.06	4	0.12
4	0.11	16	0.44
5	0.13	25	0.65
8	0.22	64	1.76
$\sum_{i=1}^{5} x_i = 21$	$\sum_{i=1}^{5} y_i = 0.57$	$\sum_{i=1}^{5} x_i^2 = 113$	$\sum_{i=1}^{n} x_i y_i = 307$

$$a=\frac{5\cdot 3.07-21\cdot 0.57}{5\cdot 113-21^2}=0.027; \qquad b=\frac{113\cdot 0.57-21\cdot 3.07}{5\cdot 113-21^2}=-0.00048.$$

The required equation of a smoothing straight line is $\bar{y}_x = ax + b$:

$\bar{y}_x = 0.027x - 0.00048$.

The graph of the required smoothing straight line is resulted in fig. 3.

Linear correlation and its characteristics.

Establishment **of force and strength** (degree of disorder of the points) of correlation relationship makes a problem *of the correlation analysis*, and *the regression analysis* establishes **the form** of dependence between X and Y (linear, curvilinear) and also allows to predict one variable on another.

For characteristic **of the form** of the equation of relationship *first of all* it is necessary to take into account theoretical reasons of character of relationship between considered variables. *Second*, character of arrangement of the points of a scatter diagram allows doing conclusions about the form of relationship too. The extended form of a scatter diagram and the angle with axes of the graph close to 45° specifies presence of correlation relationship (fig. 5 d, e). If the congestion of points forms a circle or an ellipse which long axis is parallel to one of axes of coordinates, it is possible to assume that relationships between variables is absent (fig. 5a).

Force of relationship between X and Y expresses coefficient "a" (slope) (see the formula (4)), it is named **coefficient of regress** (it frequently designate by ρ_{yx}). The coefficient of regress ρ_{yx} shows on how many units will change on the average Y, if change of X will take place on unit. More ρ_{yx}, the relationship is stronger. The linear equation of regress can be written down in the standard form

$$y - \bar{y} = \rho_{yx}(x - \bar{x}), \qquad (6)$$

Strength of relationship (degree of disorder of points) is estimated with help **of coefficient of correlation r**. We shall receive the working equation for r.

At first sight, for the characteristic of disorder of the points it is possible to count up *product* $(x_i - \bar{x})(y_i - \bar{y})$ (on fig. 4 this product is expressed by the shaded rectangular) and then to find *average value* of all products (for elimination of dependence on number of pairs of data):

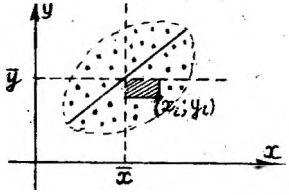


Fig. 4

$$C = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n} \qquad (7)$$

C is called **covariation** that means « connected variation ». However C depends on the scale chosen on axes. This defect of covariation can be removed if to divide C on product of standard deviations $\sigma_x \cdot \sigma_y$. In result we shall receive the characteristic of strengths of relationship – **coefficient of correlation r**:

$$r = \frac{C}{\sigma_x \sigma_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n\sigma_x \sigma_y} \qquad (8)$$

Let's open brackets in numerator and we shall take into account, that $\sum_{i=1}^{n} x_i = n\bar{x}$, $\sum_{i=1}^{n} y_i = n\bar{y}$, then

$$C = \frac{\sum_{i=1}^{n} x_i y_i - \bar{x}\sum_{i=1}^{n} y_i - \bar{y}\sum_{i=1}^{n} x_i + n\bar{x}\bar{y}}{n} = \frac{\sum_{i=1}^{n} x_i y_i - \bar{x}n\bar{y} - \bar{y}n\bar{x} + n\bar{x}\bar{y}}{n} = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{n} = \overline{xy} - \bar{x}\bar{y}, \qquad (9)$$

where $\frac{\sum_{i=1}^{n} x_i y_i}{n} = \overline{xy}$, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$. (10)

Then the coefficient of correlation r is equal

$$r = \frac{\overline{xy} - \bar{x}\bar{y}}{\sigma_x \sigma_y} \qquad (11)$$

In practice we have the data not about all general population, but only about those variables that are received from experiment (sample). Therefore determine ***sample correlation coefficient*** r_s, approximately equal to general coefficient of correlation r. Designating standard deviations for sample by s_x and s_y, we shall receive

$$r_s = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{s_x s_y} = \frac{\overline{xy} - \bar{x}\bar{y}}{\sqrt{\overline{x^2} - (\bar{x})^2}\sqrt{\overline{y^2} - (\bar{y})^2}}, \qquad (12)$$

where $s_x = \sqrt{\overline{x^2} - (\bar{x})^2}$, $s_y = \sqrt{\overline{y^2} - (\bar{y})^2}$, $\overline{x^2} = \frac{\sum_{i=1}^{n} x_i^2}{n}$, $\overline{y^2} = \frac{\sum_{i=1}^{n} y_i^2}{n}$. (13)

Properties of correlation coefficient.

1) The value of correlation coefficient changes from -1 up to +1, that is

$$-1 \le r \le 1.$$

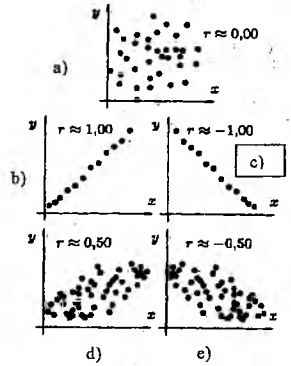


Fig. 5

2) The closer $|r|$ to 1, more closely relationship, the closer to a straight line points (fig. 5b, c) are grouped. The following gradation of strength of linear correlation relationship is accepted:

Strength of relationship	Correlation coefficient r
Relationship is absent	0
Relationship is weak	from 0 up to 0.3
Moderate	From 0.3 up to 0.7
Strong	From 0.7 up to 1
Functional	1

3) The mark of correlation coefficient shows a direction of relationship: direct (positive – fig. 5b, d) and inverse (negative, fig. 5 c, e).

Between coefficient of regress ρ_{yx} and correlation coefficient r there is a close relationship: $\rho_{yx}=r\frac{s_y}{s_x}$, therefore $b=\bar{y}-\rho_{yx}\bar{x}$ (14)

Then **predicted value y** at the given value x is equal:

$$y(x)=ax+b=r\frac{s_y}{s_x}x+(\bar{y}-r\frac{s_y}{s_x}\bar{x}) \qquad (15)$$

Check of significance of a correlation coefficient.
As r_s is determined according to sample as against correlation coefficient *of all* general population, r_s is *random value*. If $r_s\neq 0$, there is a question: whether it speaks really existing linear relationship between X and Y or is caused by random factors. For the answer to this question the value of $t_{experim}$ is calculated:

$$t_{experim}=\frac{r_s\sqrt{n-2}}{\sqrt{1-r_s^2}}. \qquad (16)$$

Further under the table (see appendix) we find value of $t_{critical}$, which has Student's distribution at the set significance value p (connected with confidential probability by the parity $p=1-\gamma$) and at number of degrees of freedom $f=n-2$. Then we compare $t_{experim}$ and $t_{critical}$: **if $|t_{experim}| > t_{critical}$, that is possible to draw a conclusion, that the correlation coefficient is *significant***, otherwise linear relationship can be caused by random factors. If the correlation coefficient appears significant it is possible to *predict* value of Y at any value X.

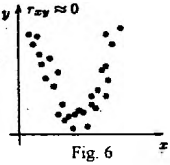


Fig. 6

Remark.
The correlation coefficient characterizes ***relationships*** between variables, ***but does not explain it***. Presence of correlation between X and Y can be caused by that: variable X influences on Y; variable Y influences on X; on X and Y the third latent factor influences that creates impression of relationship

between X and Y (spurious correlation). Besides, if r = 0 it not always speaks about absence of statistical relationship between X and Y: relationship can be nonlinear (fig. 6).

Example 2. In experiment on 13 cats the data about intrascleral (x) and intraocular pressure (y) are received:

x	19.8	7.8	12.7	13.4	10.3	13.7	16.2	15.4	21.5	8.1	11.7	7.6	6.1
y	32.5	16.1	21.3	26.8	23.4	19.7	22.9	22.2	22.6	17.6	14.3	18.6	21.4

Task:

1) Establish, is a correlation between x and y; determine correlation coefficient r_s.
2) Determine strength of correlation relationship.
3) Check up the significance of correlation coefficient.
4) Form the equation of regress and find predicated value for y at x=18.

Example 2

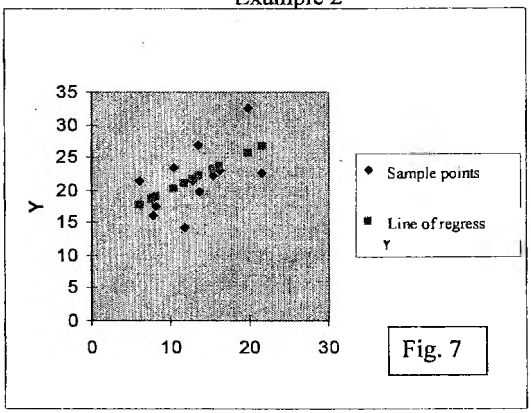


Fig. 7

Solution.

1) Let's construct the graph, having postponed along axis absciss X value of intrascleral pressure x and along axis Y value of intraocular pressure y. Then to each pair of values x and y on the graph corresponds the certain point (see fig. 7). On character of position of points it is possible to assume existence of linear correlation relationship between x and y.

Let's calculate sample coefficient of linear correlation r_s under the formula (12) with the account of (13) and (10):

$$\bar{x} = \frac{1}{13}(19.8 + 7.8 + ... + 6.1) = 12.64 \qquad \bar{y} = \frac{1}{13}(32.5 + 16.1 + ... + 21.4) = 21.49$$

$$\overline{x^2} = \frac{1}{13}(19.8^2 + 7.8^2 + ... + 6.1^2) = 180.5 \qquad \overline{y^2} = \frac{1}{13}(32.5^2 + 16.1^2 + ... + 21.4^2) = 482.2$$

$$\overline{xy} = \frac{1}{13}(19.8 \cdot 32.5 + 7.8 \cdot 16.1 + ... + 6.1 \cdot 21.4) = 283.9$$

$$s_x = \sqrt{180.5 - 12.64^2} = 4.55 \qquad s_y = \sqrt{482.2 - 21.49^2} = 4.51$$

$$r_s = \frac{283.9 - 12.64 \cdot 21.49}{4.55 \cdot 4.51} = 0.598$$

2) Using the table of gradation of r, we judge: relationship of x and y is moderate, positive.

3) For check of significance of correlation coefficient r, we shall calculate under the formula (16) value of $t_{experim}$:

$$t_{experim}=\frac{0.598\cdot\sqrt{13-2}}{\sqrt{1-0.598^2}}=2.47$$

Under the table we find value of $t_{critical}(11; 0.05)=2.20$. As $|t_{experim}| > t_{critical}$, that is $2.47 > 2.20$, that we judge, that the correlation coefficient is significant.

4) By the formula (14) we find coefficient of regress:

$$\rho_{yx}=0.595\frac{4.51}{4.55}=0.589.$$

Further, substituting ρ_{yx} in the formula (6) and calculating b, we find the equation of regress $y = \rho_{yx}x + b$:

$$y=0.589\,x + 14.$$

Coefficients **a** and **b** could be found under formulas (4) and (5) also.

At last, we calculate predicated value at x=18:

$$y(18)= 0.589\cdot 18 + 14 = 24.6$$

The **coefficient of determination r^2** is defined as the percent of the variation in the values of the dependent variable y that can be explained by variation in the value of the dependent variable x. In our example $\mathbf{r^2=0.598^2= 0.36}$ or 36%, which means that 36% of total variation in y can be explained by the linear relationship between x and y (as described by the regression equation). The other 64% of the total variation in y remains unexplained. Roughly speaking, $\mathbf{r^2}$ tells how many of the points of data fall within the results of the line formed by the regression equation. $0 \le \mathbf{r^2} \le \mathbf{1}$, values of 1 or 0 would indicate the regression line represents all or none of the data, respectively. A higher coefficient is indicator of a better goodness of fit for the observations.

Steps of the work

Using the experimental data (under the instruction of the teacher) carry out the following task:

At eight men have been measured height (x) and weight (M=y):

X (cm)	165	176	175	168	167	172	175	180
M (kg)	56	75	70	61	62	63	72	80

Task:

1) Establish is any correlation between x and y; determine correlation coefficient r_s.
2) Determine strength of correlation relationship.
3) Check up the significance of correlation coefficient.
4) Form an equation of regress $y = \rho_{yx}x + b$, construct the graph, find the determination coefficient r^2.

Control questions

1) What is correlation relationship between X and Y?
2) What is the equation of regress, a line of regress?

3) What we can find with the help of the method of the least squares? Write down working formulas for $a$and b.
4) What problems are solved correlation and regression analysis?
5) How does estimate the form of relationship? How does find coefficient of linear regress ρ_{yx} (two ways) and y-intercept b?
6) What does correlation coefficient estimate? Write down the working formula for correlation coefficient and its properties.
7) Testing the significance of correlation coefficient.
8) How does find predicted value of y at given value x?
9) What is determination coefficient? What is mean?
10) Copy examples 1 and 2, similarly solve the offered task.

Laboratory work №6

Use of the Spearmen's rank coefficient of correlation in medicine

Purpose of the work: to familiarize with the Spearmen's rank correlation.
Equipment: statistical tables.

Theory of the work

Correlation analysis is studied by measurement of degree of relationship between two variables *x* and *y*, for example: "factor-response", "dose-effect", etc. However in many of cases *x* and *y* have not quantitative, but qualitative characteristics. For example, we take from some cell culture a series of samples and we will concern with the problem: whether the sampling order influences number of bacteria containing in a sample. In this case the variable *x* (the sampling order) represents the regulated set of numbers: 1st sample, 2nd sample etc. It is the ordinal variable. The number of bacteria (variable y) can be calculated. So, all range of *x* or y we can regulate on qualitative characteristic, i.e. arrange. The rank is the order number of value of the ordinal random variable arranged in ascending or decreasing order of their magnitudes, for example: 1st place, 2d place, 3d place etc. Such ordinal scales quite often meet in clinical trials and also at estimation of various psychological tests. In this case is computed the **rank coefficient of correlation of Spearmen**, it is *nonparametric analogue of the Pearson's coefficient of correlation.*

So, we compute the Spearmen's coefficient of correlation (r_s or ρ) in the following cases: 1) at least one of variables *x* or *y* is measured in rank (ordinal) scale; 2) distribution at least one of variables does not submit to the normal law; 3) connection between *x* and *y* is non-linear (but monotonous); 4) the size of sample is small. It is necessary to have in view that at passage from concrete numbers to ranks part of information in the sample is lost, therefore rank test is more rare reject a null hypothesis than parametric (Pearson's test is more powerful than Spearmen's).

The formula for calculation of Spearmen's ρ (r_s) is gained from the formula of the coefficient of correlation of Pearson, if in it instead of x_i and y_i to substitute their ranks:

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)} \qquad (1),$$

Where $d_i = x_i - y_i$ is the difference of ranks of i -th pair of value x_i and y_i.

Properties and characteristics of Spearmen's ρ (r_s): 1) r_s has limits $-1 \le r_s \le 1$; if ranks are equal for all x_i and y_i, then all $d_i = 0$ and $r_s = 1$. If ranks x_i and y_i are arranged upside-down then $r_s = -1$. In both cases there is a functional connection between *x* and *y* and this function (non-linear association) is not certainly linear and can be any monotonic increasing or diminishing function. If

between x and y there is not a functional connection, the less r_s differs from zero, then it is less and a connection degree between x and y.

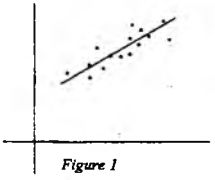


Figure 1

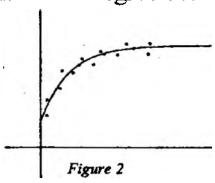
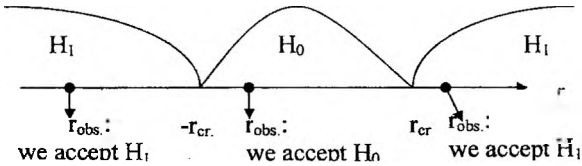


Figure 2

2) Value of coefficient of determination r_s^2 is not computed, since r_s^2 does not represent a share of variation *of y* that it is possible to explain by changing of x.

3) Testing H_0: $\rho_s = 0$ (there is no correlation) at $n \leq 50$ value of $r_{obs.}$ is computed by formula (1) and value r_{crit} is possible to find under the tables (tab. 1).

Fig. 3



If $|r_{obs.}| < r_{cr}$ H_0 is accepted and if $|r_{obs.}| > r_{cr}$ H_1 is accepted (fig. 3). If the sample size is more 50 it is necessary to use Student's t-distribution with n-2 degree of freedoms:

$$t_{obs.} = \frac{r_{obs.}\sqrt{n-2}}{\sqrt{1-r_{obs.}^2}},$$

value of $t_{cr.}$ we can find under the table of critical points of the Student's distribution. If $|t_{obs.}| < t_{cr.}$ the null hypothesis is accepted, if $|t_{obs.}| > t_{cr.}$ H_1 is accepted.

We will consider procedure of determination Spearmen's r_s in hand-operated and machine alternative on the following example.

Example 1.

Researcher, studying sleep physiology at depression has come to necessity of estimation of severity of depression. The Beck' scale of depression is grounded on a simple questionnaire in the application filled with the patient. Application of the Hamilton's scale of depression is more difficult, because demands participation of the doctor, but gives more exact result. Having compared estimates on both scales for 10 patients, the author has gained following outcomes (see the table, lines 1 – 3). How much two estimations be in accord with two scales? Let significance level is α=0.05.

Solution ("hand-operated" case).

1. We convert the table data into ranks R_x and R_y as shown in the table. To equal values of x_i or y_i we assignee the ranks equal to average arithmetical.
2. We compute d_i and d_i^2.
3. By formula (1) we compute $r_s = 1 - \frac{6 \cdot 19.5}{10(10^2-1)} = 0.8819$.
4. We state hypotheses: H_0: $\rho = 0$ (ρ is the population correlation coefficient); H_1: $\rho \neq 0$; we find $r_{obs.} = r_s = 0.8819$; value r_{cr} (10; 0.05) = 0.648 we took from the table of Spearmen coefficients of correlation, as $|r_{obs.}| > r_{cr.}$ H_1 is accepted: there is strong

positive correlation between the scales depression of Beck and Hamilton on the significance level of 0.05.

№ of patient	1	2	3	4	5	6	7	8	9	10	
Estimation on the Beck's scale	20	11	13	22	37	27	14	20	37	20	
Estimation on the Hamilton scale	22	14	10	17	31	22	12	19	29	15	
R_x	5	1	2	7	9.5	8	3	5	9,5	5	
R_y	7.5	3	1	5	10	7.5	2	6	9	4	
$d_i = R_x - R_y$	- 2.5	- 2	1	2	- 0.5	0.5	1	- 1	0.5	1	$\sum d_i = 0$
d_i^2	6.25	4	1	4	0.25	0.25	1	1	0.25	1	$\sum d_i^2 = 19.5$

Solution (machine alternative: the package Statistica, v6).

1. We enter data in the columns Var 1 and Var 2.
2. We choose the module Statistica → the Nonparametric data → Correlations (Spearman, ...) → OK.
3. In the opened dialogue window of nonparametric correlations we select: 1) advanced; 2) variables: 1-st var. list: var1, 2-nd var. list: var2; 3) p-level: 0.05 → Spearman rank R.
4. We read out outcome: coefficient of correlation (Spearman R):0.882783, p-level: p = 0.000715 ($p<0.05$).

As we see, the outcomes in hand-operated and machine alternative have completely coincided.

Example 2.

5 students have the following ranking in Physics and Anatomy subject. Is a correlation (association) between the rankings in Physics and Anatomy subject?

Solution (see the table).

$r_{obs} = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \cdot 30}{5(5^2-1)} = -0.5$. H_0: $\rho = 0$, H_1: $\rho \neq 0$. $r_{cr}(5; 0.05) = 1$. As $|r_{obs}| <$ r_{cr} H_0 is accepted. Conclusion: there is no any correlation between the rankings in Physics and Anatomy subject.

Student	Ashley	David	Owen	Steven	Frank	
Physics class Rank, R_x	1	2	3	4	5	
Anatomy class Rank R_y	5	3	1	4	2	
$d_i = R_x - R_y$	-4	-1	2	0	3	
d_i^2	16	1	4	0	9	$\sum d_i^2 = 30$

Example 3.

Find the critical value t_{cr} of the Spearmen's rank correlation coefficient r_s when the data consists of 60 pairs of ranks. Assume a two-tailed case with the 0.05 significance level.

Solution. Because n exceeds 50, we will use Student's t-distribution with n-2=58 degree of freedoms: $t_{cr}(58;0.05)=2.002$.

Thus, to the advantages of the Spearmen's rho it is possible to refer simplicity of calculation and fast way of rejection of the null hypothesis. Besides, nonparametric methods can be applied safely also in case of the normal distribution, however its sensitivity will be a little below sensitivity of parametric methods. Sensitivity of the Spearmen's coefficient of correlation is approximately 0.9 from the Pearson's correlation coefficient r, that is loss is very insignificant (so, Pearson's r according to example 1 computed by Statistica 6 is equal to 0.939, and Spearmen's ρ: $r_s=0.883$, that is close to each other).

Execution of the work

Solve the next problems, using the Spearmen's rank correlation:

a) calculate the rank correlation coefficient r_s for the given sample data;

b) find the critical value of r_s at significance level of 0.05;

c) based on the results of parts (a) and (b), decide whether there is significant positive or negative correlation, or no significant correlation.

1. A medical researcher test the effects of a drug on the time it takes a patient to perform a standard manual task. The drug amounts in mg and corresponding times in seconds for randomly selected patients are given in the table:

Drug amount	15	20	25	30	35	40	45	50	55	60	65	70	75
Time	48	46	55	54	60	58	73	74	82	90	105	130	200

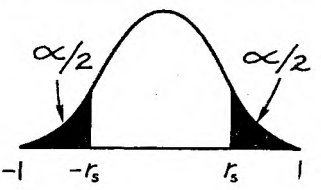
2. A medical researcher study correlation between industrial work record (in years) and the number of industrial accidents:

Industrial work Record (years)	number of industrial accidents
<1	24
1-2	16
2-4	12
4-6	12
>6	6

3. Find the correlation between ranks of two subjects for 5 students of your group.

Control questions

1. What is the rank of a variable? Give the examples of ranking.
2. When do we compute the Spearmen's coefficient of correlation?
3. What is the formula of the test statistic of the Spearmen's rho?
4. What are the advantages of the Spearmen's rho? Copy all examples.



Critical Values of Spearman's Rank Correlation Coefficient r_s

n	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$
5	.900	—	—	—
6	.829	.886	.943	—
7	.714	.786	.893	—
8	.643	.738	.833	.881
9	.600	.683	.783	.833
10	.564	.648	.745	.794
11	.523	.623	.736	.818
12	.497	.591	.703	.780
13	.475	.566	.673	.745
14	.457	.545	.646	.716
15	.441	.525	.623	.689
16	.425	.507	.601	.666
17	.412	.490	.582	.645
18	.399	.476	.564	.625
19	.388	.462	.549	.608
20	.377	.450	.534	.591
21	.368	.438	.521	.576
22	.359	.428	.508	.562
23	.351	.418	.496	.549
24	.343	.409	.485	.537
25	.336	.400	.475	.526
26	.329	.392	.465	.515
27	.323	.385	.456	.505
28	.317	.377	.448	.496
29	.311	.370	.440	.487
30	.305	.364	.432	.478

For $n > 30$ use $r_s = \pm z/\sqrt{n-1}$, where z corresponds to the level of significance. For example, if $\alpha = 0.05$, then $z = 1.96$.

To test H_0: $\rho_s = 0$

against H_1: $\rho_s \neq 0$

LABORATORY WORK №7

Probability models in medicine: conditional probability; composite probability formula; Bayes's theorem

Purpose of the work: to familiarize with one the most used probability models in medicine.

Devices and accessories: calculator.

Theory of the work

Conditional probability of an event A is a probability of this event A calculated at condition that the event B has already happened. Conditional probability is denoted by P_B (A) or P (A/B).

Example: in a box there are 10 tablets: 3 are white and 7 are dark blue. 2 tablets are taken one by one. Determine probability: 1) that the second tablet will be white if the dark blue tablet was the first; 2) that the first tablet was dark blue and second was white.

Solution: 1) the probability of event B (the dark blue tablet is taken) is equal:

$$P(B) = \frac{7}{10}.$$

The probability that the second tablet will be white provided that 1st tablet was dark blue (conditional probability) is equal:

$$P(A/B) = \frac{3}{9}.$$

2) The probability that simultaneously the first tablet was dark blue and second was white is equal: $P(B \cdot A) = P(B) \cdot P(A/B) = \frac{7}{10} \cdot \frac{3}{9} = \frac{7}{30}$.

The composite probability formula is generalization of theorems of addition and multiplication of probabilities. The formula is applied to a problem solution on determination of probability of event A that can happen with one of incompatible events H_i organizing a complete group. Thus events H_i are known as hypotheses. Then the probability of occurrence of A is defined **by the composite probability formula**:

$$P(A) = \sum_{i=1}^{n} P(H_i) \cdot P(A/H_i)$$

Example (ancient problem): the blind aged man has gone out a point B to the point A without a guide. What is probability that he will come to the point A? (see fig.1)

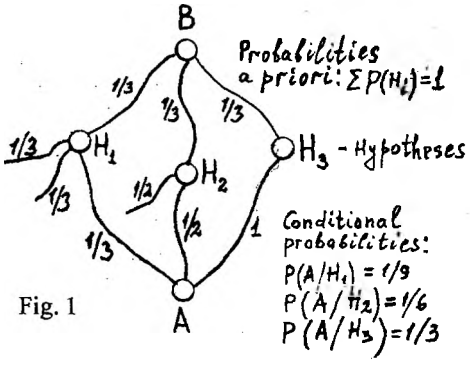


Fig. 1

Solution: as it is supposed the aged man in a random way goes out on a road, then probability to select each of intermediate points H_1, H_2 and H_3 is equal to $\frac{1}{3}$. $\sum P(H_{ii}) \cdot P(A/H_i)$ is a **composite** probability, P (A/H_i) are the **conditional** (joint) probabilities. But further from each point H_i he comes to the point A with different probability: for the point H_1 this probability is equal 1/3, for the point H_2 is equal $\frac{1}{2}$, for the H_3 probability is equal 1 (only one road). Then the composite probability to hit in the point A:

$$P(A) = P(H_1) \cdot P(A/H_1) + P(H_2) \cdot P(A/H_2) + P(H_3) \cdot P(A/H_3) = \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1 = \frac{11}{18}.$$

Probabilities P (H_i) are known **as a priori probabilities** (probabilities *before* a trial). The information on these probabilities in clinical practice can be taken for example from the case record, experience of the doctor, the demographic data etc. The sum of all a priori probabilities should be equal to 1: $\sum P(H_{ii}) = 1$.

The scheme on fig. 1 is named a *tree of probabilities.*

In practice often it is necessary to make *overestimation* of some initial hypotheses. So, the doctor, having examined the patient has made the *a priori* supposition about probabilities of several diseases, using the case record, personal experience. After deriving of outcomes of analyses, doctor does an improvement of the initial hypotheses, some initial suppositions are thrown, and the final diagnosis is made. So, the **Bayes's theorem** answers a problem: how probabilities of hypotheses H_1 ... H_i have changed after event A has happened:

$$P(H_i/A) = \frac{P(H_i)}{\sum P(H_i) \cdot P(A/H_i)} \cdot P(A/H_i).$$

Let, for example, in a condition of the previous problem it became known, that the aged man has came to A. What is the probability that it went through the point H_1? H_2? H_3? Using the theorem of Bayes, we have:

$$P(H_1/A)=\frac{P(H_1)\cdot P(A/H_1)}{P(A)}=\frac{\frac{1}{3}\cdot\frac{1}{3}}{\frac{11}{18}}=\frac{2}{11}$$

$$P(H_2/A)=\frac{P(H_2)\cdot P(A/H_2)}{P(A)}=\frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{11}{18}}=\frac{3}{11}$$

$$P(H_3/A)=\frac{P(H_3)P(A/H_3)}{P(A)}=\frac{\frac{1}{3}\cdot 1}{\frac{11}{18}}=\frac{6}{11}.$$

APOSTERIORI PROBABILITIES

So, the Bayes's theorem allows one to compute a conditional probability based on the available information the general scheme of the Bayesian analysis is next:

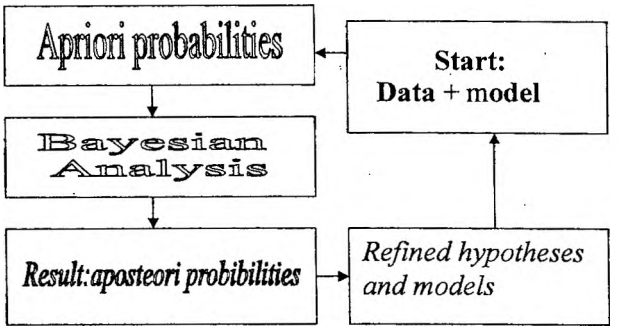


Fig. 2

Posterior (a posteriori) probabilities $P(H_i/A)$ are the probabilities received after a trial on the basis of a priori probabilities.

The Bayesian analysis is used **in medicine** for 1) the analysis of efficiency of diagnostic tests; 2) at the clinical analysis of solutions and construction of diagnostic algorithms; 3) for the analysis medicolegal data, etc.

Execution of the work

Using the Byes's theorem, solve the following problems:

1. A doctor believes that a patient has a 10% chance of having Lyme disease. He gives a patient a blood test and the test comes out positive. The manual for this test says that out of 100 patients with Lyme disease, 80% test positive. Moreover, out of 100 patients with the Lyme disease 30% test positive. What is the probability that the patient has Lyme disease? Construct a tree of probabilities.

2. Let a diagnostic test of some disease gives the positive response with probability of 0.95 if at examined there is the disease, and probability of 0.05, if at examined disease is not present. Let in the group examined probability to have this disease is equal 0.001 (1st subgroup), and probability not to have this disease is 0.999 (2nd subgroup). What is probability that examined having a positive response on the test is really subject to the given disease (see the fig.3)? Construct a tree of probabilities.

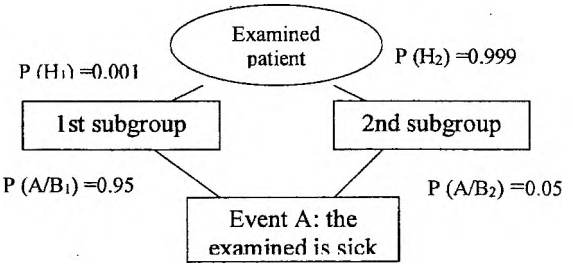


Fig. 3

3. According to statistics, smoking men at age over 40 years died from the cancer of lungs in 10 times more often than non-smoking person. Let 60 % of men are smoking. What is the probability that examined taken man will die of the cancer of lungs? What is the probability that the man who has died of a cancer of lungs smoked? Did not smoke? Construct a tree of probabilities. Compare a priori and posterior probabilities.

Control questions

1) What is the conditional probability?
2) What we can determine with help of the composite probability formula.
3) Explain destination of the Bayes's theorem.

LABORATORY WORK №8

DETERMINATION OF THE MOMENT OF INERTIA OF A BODY AND CHECK OF THE BASIC LAW OF DYNAMICS OF THE ROTATIONAL MOTION

Purpose of the work: to determine a moment of inertia of the Oberbek`s pendulum, to study dependence of angular acceleration on the moment of inertia at the constant moment of force.

Devices and accessories: Oberbek`s pendulum, two-meter ruler, stop watch, vernier calipers, cord of length 2.5 m, load of mass 200 g and 100 g.

Theory of the work

Moment of inertia I_i of a material point of mass Δm_i taking place on the distance r_i from the axis of rotation is numerically equal to product of mass of the material point and the square of distance of the point from the axis, i. e. $I_i = \Delta m_i r_i^2$ (fig.1).

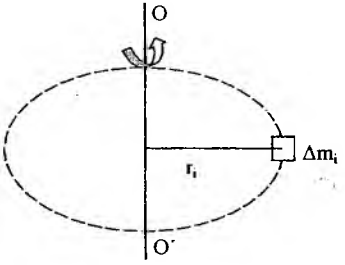


Fig. 1

The body can be presented consisting of n the same elementary mass. Then the moment of inertia of the body:

$$I = \sum_{i=1}^{n} \Delta m_i \cdot r_i^2 .$$

Unit of measurement of the moment of inertia in SI: $[I] = \text{кг} \cdot \text{м}^2$. Rotation of a body around of an axis is caused by the rotating moment or simply by the moment of force.

Moment of force M concerning an axis of rotation is the vector quantity numerically equal to product of force F and length of the perpendicular d (*arm of the force*) lowered from the center of rotation on the direction of action of the force. The module of the moment of force (fig. 2):

$M = F \cdot d = F \cdot r \sin \alpha$.

Direction of the vector $\vec{M}$ coincides with the direction of motion of right screw at its rotation from $\vec{r}$ to $\vec{F}$ (fig. 2).

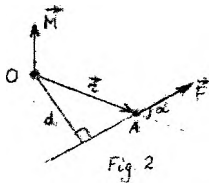


Fig. 2

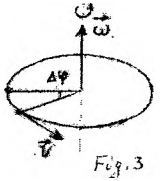


Fig. 3

If in time Δt the body rotates on the angle $\Delta\varphi$ speed of its rotation at present time is characterized *by angular speed ω (fig. 3)*:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\varphi}{\Delta t} = \frac{d\varphi}{dt}.$$

Under action of the moment of the force the rigid body fixed on the axis gets *angular acceleration β*:

$$\beta = \frac{d\omega}{dt},$$

where ω is angular speed.

Dependence of angular acceleration β of a rotating body on the moment of force M working on a body and the moment of inertia I of the body concerning of axis around of which there is a rotation is defined ***by the basic equation (law) of dynamics of rotational motion***:

$$\boxed{M = I\beta = I\frac{d\omega}{dt}}.$$

The formula of the law for rotary motion is *similar to the formula of Newton's 2-nd law* for forward movement:

$$F = ma.$$

To the force F corresponds the moment of force M; to acceleration – angular acceleration β; to mass m – the moment of inertia I. Just as the mass m characterizes inertial properties of a body at the forward movement, the moment of inertia I characterize inertial properties of bodies at rotary movement.

Knowledge of the moment of inertia of bodies and the basic law of dynamic of rotary movement is necessary in many areas of science and technics. In some sections of space and sports medicine, orthopedy there is a necessity of measurement of the moment of inertia of a human body. Moment of inertia at rotary movement of a human trunk or his finitenesses can be calculated approximately with help of the formulas for the moment of inertia of the cylinder and a round core or from experiment. In molecular biology are determined the moments of inertia of complex molecules.

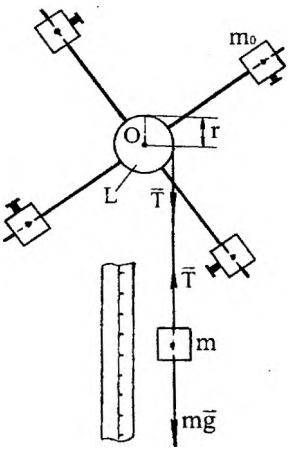


Fig.4

Description of installation

The moment of inertia of a body can be determined from the basic law of dynamic of rotary movement:

$$I = \frac{M}{\beta}. \quad (1)$$

For measurement of the moment of force working on the body and the angular acceleration imparted to this body can be applied crosswise Oberbek's pendulum (fig. 4).

The device consist of pulley L of radius r fixed on axis O, four cores located under the angle 90° to each other and four identical cylindrical loads m_0, which can be moved along cores with fixing on the certain distances.

Loads are fixed symmetrically, i.e. so that the centre of gravity of a pendulum is on the axis of rotation.

Device is resulted in rotary movement by a load of mass m. The load is attached to the end of the cord which has been reeled up on the pulley. The moment of forces of friction is much less than the rotating moment ($M_{fr} << M$).

If the load suspended on the string falls from height h in time t, then $h = \frac{at^2}{2}$, where a is linear acceleration on the rim of the pulley. Then:

$$a = 2h/t^2 . \qquad (2)$$

Thus the pulley with cores and the loads located on it will rotate with angular acceleration β:

$$\beta = a/r . \qquad (3)$$

From (2) and (3) follows that:

$$\beta = 2h/rt^2 . \qquad (4)$$

Rotating moment we shall find under the formula: $M = T \cdot r$, where T is the force of the tension of the string, r is the arm of this force.

Force of tension of the string we shall find from the II law of Newton for the load of mass m:

$$ma = \sum_{i=1}^{n} F_i , \text{ but } ma = mg - T, \text{ whence } T = mg - ma.$$

Then: $$M = (mg - ma) \cdot r = m\left(g - \frac{2h}{t^2}\right) \cdot r. \qquad (5)$$

Having substituted in the formula (1) formulas (4) and (5) we shall receive:

$$I = \frac{mt^2r^2\left(g - \frac{2h}{t^2}\right)}{2h}. \qquad (6)$$

For determination of the moment of inertia I is necessary to determine from experiment all values in the right part of the formula (6).

Order of the work

1. Determination of a moment of inertia of the pendulum.

1. Move loads to ends of cores and fix their by screws on last divisions of the cores. Thus the pendulum should not rotate, if the system is correctly balanced.
2. Reel up in regular intervals the string on the pulley. On the ruler put two labels on the distance corresponding to height of fall h.
3. With the help of vernier callipers determine radius of the pulley.
4. Having given opportunity to the load m to fall, with the help of the stop watch determine time of falling t between two labels.
5. Experiment iterates for two loads: 200g and 100g.
   Data bring in the table.

№	t, s	M, N·m	β , s^{-2}	I, кg·m^2	Constants
1					m =0.2 kg;
2					0.3 kg
3					r =
4					h=
					g=9.81 m/c^2

II. Examination of laws of the rotary motion.

1. Sequentially anchoring loads on the divisions marked on rods, discover time t (m=const) for two positions of loads.
2. Under formulas (5), (4) and (1) determine the moment of force, angular acceleration and the moment of inertia of the pendulum for each case.

***Conclusion**: according to the basic equation of dynamics of rotary motion with growth of the moment of inertia I angular acceleration β decreases that is β ~1/I.*

Control questions

1. Define I, M, β; its units.
2. In what areas of medicine the knowledge of the moments of inertia of bodies is necessary?
3. Formulate and write down the basic equation of dynamics of rotary motion. Compare it with the II law of Newton for forward motion.
4. Deduce working formulas for determination of I, M, β.

Laboratory work № 9

Study of oscillatory motion with help of kymograph

Purpose of the work: 1) to study the theory of the basic kinds of oscillations;

2) to make the record and to generate the equation of damped oscillations and beats.

Equipment: 1) kimograph, special tape for record of oscillations;

2) induction coil; a source of direct current of 12 V;

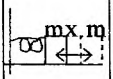
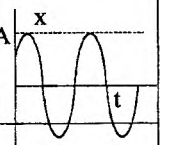
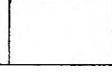
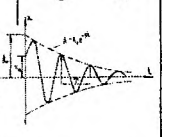
3) stop watch.

Theory of the work

Oscillations are called motions which repeat through the certain time intervals. Oscillations play the big role in the biomechanics of a human organism.

Some kinds of oscillations (see the table 1):

Table 1

Kind of oscillation	Phisical model of oscil-s	Working forces (agents)	Differential equation of oscillation	Solution of the differential equation	Graph of oscillations
Harmo nious oscillations		$F_{fr}=0$ $F_{el} \neq 0$ $ma=F_{el}+F_{fr}$	$\frac{d^2x}{dt^2}+\omega_0^2 x=0,$ $a=\frac{d^2x}{dt^2};\ \frac{k}{m}=\omega_0^2$	$x=A\sin(\omega_0 t+\varphi_0)$ x is displacement; A is amplitude; ω_0 - frequency; $(\omega_0 t+\varphi_0)$ is phase	
Damped oscillations		$F_{fr}\neq 0$ $F_{el} \neq 0$ $ma=F_{el}+F_{fr}$	$\frac{d^2x}{dt^2}+2\beta\frac{dx}{dt}+\omega_0^2 x=0$ $\frac{k}{m}=\omega_0^2;\ \frac{r}{m}=2\beta$	$x=A_0 e^{-\beta}\sin(\omega t+\varphi_0)$ A_0- initial amplitude; β- coefficient of attenuation, φ_0-initial phase	

Damped oscillations

From the graph of oscillations is visible, that oscillations occurs under the harmonious law with amplitude $A = A_0e^{-\beta t}$ and A decreases on exponent law. Coefficient β (coefficient of attenuation) shows as quickly the amplitude of oscillations decreases. Coefficient β is numerically equal to value, inversely

proportional to a time interval $t_{A/e}$ for which the amplitude decreases in e≈2,718… times: $\beta = \frac{1}{t_{A/e}}$.

Except of β in practice degree of attenuation is characterized **logarithmic decrement of attenuation λ.**

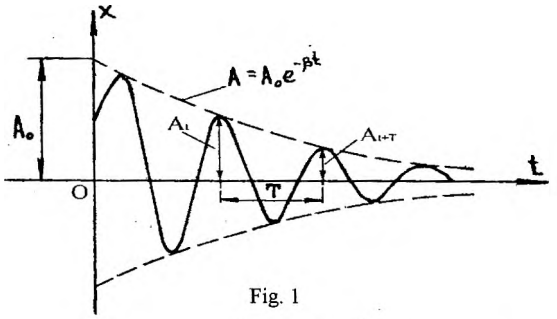


Fig. 1

λ is the value equal to the natural logarithm of the ratio of two consecutive amplitudes, divided by the interval of time equal to the period of oscillations:

$$\lambda = \ln\frac{A_t}{A_{t+T}} = \ln\frac{A_0 e^{-\beta t}}{A_0 e^{-\beta(t+T)}} = \ln e^{\beta t} = \beta T \qquad (1)$$

From (1) it is possible to receive the practical formula for calculation of β:

$$\ln\frac{A_t}{A_{t+nT}} = \ln\frac{A_0 e^{-\beta t}}{A_0 e^{-\beta(t+nT)}} = \ln e^{\beta nT} = \beta nT \Rightarrow \beta = \frac{\ln\frac{A_t}{A_{t+nT}}}{nT} \qquad (2)$$

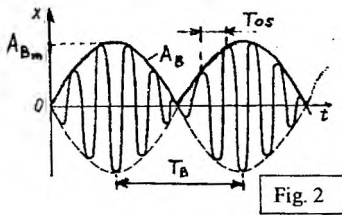


Fig. 2

Beats

Beats is a special kind of harmonious oscillations with pulsing amplitude; it is used at the analysis of hearing, at electrotherapy.

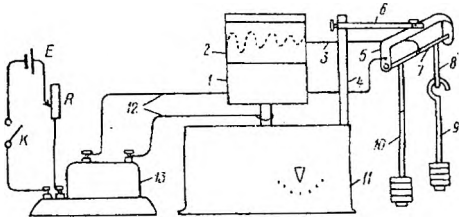
Beats are result of addition of two harmonious oscillations with close frequencies $\omega_1 \approx \omega_2$ going to the same direction: $x = x_1 + x_2 = A\cos\omega_1 t + A\cos\omega_2 t$ (for simplicity we have taken identical amplitudes $A_1 = A_2 = A$). Carrying out addition, we shall receive

$$x = A(\cos\omega_1 t + \cos\omega_2 t) = \left[2A\cos\frac{\omega_1 - \omega_2}{2}t\right]\cdot\cos\frac{\omega_1 + \omega_2}{2}t \qquad (3)$$

We have received the **equation of displacement for beats**. Apparently from (3), oscillations pass under the harmonious law with **frequency of** $\frac{\omega_1+\omega_2}{2}\approx\omega$, but thus the amplitude is not constant and changes slowly under the law $\cos\frac{\omega_1-\omega_2}{2}t$ (factor in square brackets in (3)). Since on sense, the amplitude of oscillations can not be negative variable and the amplitude of beats is equal to the module of expression in square brackets: $A_B = [2A\cos\frac{\omega_1-\omega_2}{2}t]$.

Graph of amplitude A_B of beats is shown in figure 2 by continuous line. From the diagram is visible that frequency of amplitude of beats A_B in 2 times is more than frequency of cos: $\frac{\omega_1-\omega_2}{2}\cdot 2=\omega_1-\omega_2$. From this formula follows the important conclusion: **frequency of beats (that is frequency from which pulses A_B) is equal to the difference of frequencies of the added oscillations:** $\omega_B=\omega_1-\omega_2$ or $\nu_B=\nu_1-\nu_2=\frac{\omega_1-\omega_2}{2\pi}$. Period of beats is $T_B=\frac{1}{\nu_B}=\frac{2\pi}{\omega_B}=\frac{2\pi}{\omega_1-\omega_2}$.

DESCRIPTION OF EQUIPMENT



Kymograph is intended for recording of various oscillatory processes (for example, pneumography is registration of respiratory motion). Kymograph is the rotating metal drum 1, it consists of the case 11, in which the engine is placed. The oscillatory system consists of two pendula 9, 10 fixed on the same axis. Pointer is the metal core 3 fixed on the same axis 7. Between the pointer and the metal drum is enclosed a high voltage from the source of direct current. The arising spark burns trajectory of oscillations on the paper tape 2.

Warning: in operating time it is impossible to concern the drum and the pointer of kimograph! Record is made by the teacher or the laboratorian.

Execution of the work

I. Recording of oscillations.

1. For record of damped oscillations remove a pendulum 10 on the flat spring. Fix the paper tape on the drum so, that the end of the pointer was in the middle of tape on distance of 1-2 mm.

2. Switch on consistently the source, the engine and the coil.

After recording of 11-12 periods disconnect the coil and the engine.

3. By the stop watch determine time of 10 periods of oscillations.

4. For recording of beats fix the second pendulum on the flat spring and similarly write down some periods of beats.

5. Measure time of two periods of beats T_B.

II. Generation of the equation of damped oscillations.

1. Remove the tape and lead axes x and t as on fig. 1. To find the initial phase φ_0, let's make a proportion. One period of oscillations corresponds to change of phase on 2π (segment l), initial phase φ_0 corresponds to the segment l_0, whence $\varphi_0 = \frac{2\pi \cdot l_0}{l}$.

Take the measurement of A_0 on the tape (fig.1).

2. Count up the frequency of oscillations $\omega = \frac{2\pi}{T}$, where $T = \frac{t_n}{n}$, t_n is time of n oscillations (n=10).

3. Having measured amplitudes A_t и A_{t+10T}, under the formula (2) calculate β.

4. Substitute in the equation $x = A_0 e^{-\beta t} \sin(\omega t + \varphi_0)$ the received values A_0, β, ω and φ_0 and write down the received equation. Data put in the table 2.

III. Generation of the equations of beats

1. Draw the axes x and t as in fig. 2. Take the measurement of A_B.

2. For determination of ω_1 and ω_2 it is necessary to solve system from two equations: $\begin{cases} \omega_1 - \omega_2 = \omega_B \\ \frac{\omega_1 + \omega_2}{2} = \omega_{OS} \end{cases}$

where $\omega_B = 2\pi/T_B$ is frequency of beats, $\omega_{OS} = 2\pi/T_{OS}$ is frequency of oscillations. Period of one oscillation is $T_{OS} = \frac{T_B}{m}$, where m is quantity of periods of oscillations in one period of beats (between two minima).

3. Substitute in (3) numerical values of $A = \frac{A_B}{2}$, ω_1, ω_2 and write down the equation of beats. Data put in the table 3.

Table 2

n	t_n, s	T, s	ω, s^{-1}	A_0, mm	A_t, mm	A_{t+nT}, mm	β, s^{-1}	l_0, mm	l, mm	φ_0

Table 3

A_B, mm	A, mm	T_B, s	ω_B, s^{-1}	m	T_{OS}, s	ω_{OS}, s^{-1}	ω_1, s^{-1}	ω_2, s^{-1}

Control questions

1. Write down the differential equation and its solution for damped oscillations.
2. Write down working formulas for ω, T, β, φ_0 that necessary for generation of damped oscillations.
3. Write down the equation of displacement x (t) for beats and specify frequency of oscillations and frequency of beats.
4. Write down working formulas for A, ω_1, ω_2 for beats.

LABORATORY WORK №10

STUDY OF MECHANICAL PROPERTIES OF A BONE TISSUE

Purpose of the work: to study elastic properties of a bone tissue, to determine the module of elasticity of a human bone tissue and to compare it with the module of elasticity of a metal sample.

Devices and accessories: installation for determination of the module of elasticity of samples, the sample of a bone tissue, the metal sample, a set of loads of 2 N, vernier calipers, a ruler, the indicator of small displacements.

Theory of the work

Knowledge of mechanical properties of tissues of human organism is necessary in surgery, orthopedy, traumatology, at selection of transplants for bone-plastic and reconstructive operations.

Deformation is called change of mutual position of the points of a body under action of external forces. Deformation is known as elastic if after cancellation of forces the body restores its form, otherwise deformation is known as plastic (residual).

Basic kinds of deformations: 1.stretching; 2. compression; 3. bend; 4. shift; 5. torsion (fig. 1).

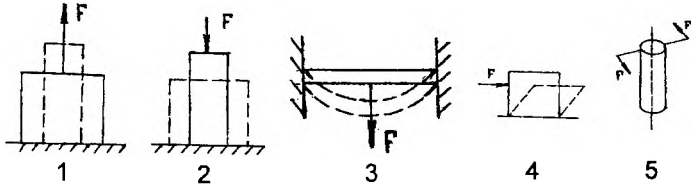


Fig. 1

In the theory of elasticity is proved, that all kinds of deformation can be shown to simultaneously occurring deformations of stretching (compression) and shift.

The quantitative measure describing degree of deformation is **percent elongation** or longitudinal strain: $\varepsilon = \frac{\Delta \ell}{\ell}$, where $l = l_1$ is initial length of a body, $\Delta \ell = \ell_2 - \ell_1$ is change of length (**absolute elongation**). At stretching $\Delta \ell > 0$, at compression $\Delta \ell < 0$.

The force working per unit of the area of cross-section is called the **pressure** (mechanical):

$$\sigma = \frac{F}{S}. \qquad (1)$$

For elastic deformations Hooke`s law is fair according to which the mechanical pressure and percent elongation are directly proportional each other:

$$\boxed{\sigma = E \cdot \varepsilon} \text{ or } \boxed{\frac{F}{S} = E \cdot \frac{\Delta \ell}{\ell}}. \qquad (2)$$

Where the coefficient of proportionality E is called the ***Young`s modulus***. From (2) is visible that *Young`s module is numerically equal to the mechanical pressure at* $\varepsilon = 1$.

$\varepsilon = 1$, when the length of the sample is doubled: $1 = \frac{l_2 - l_1}{l_1} \Rightarrow l_2 - l_1 = l_1 \Rightarrow l_2 = 2 \cdot l_1$.

Dependence of a mechanical pressure on ε for solid bodies is shown on fig. 2. Dependence $\sigma = f(\varepsilon)$ for a compact bone tissue has a similar kind. At a small pressure deformation has elastic character that is expressed on the graph by directly proportional dependence (site OB).

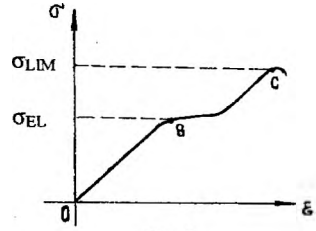


Fig. 2

The greatest mechanical pressure σ_{EL} at which deformation keeps elastic character is known as the *limit of elasticity*. At the further increase in a pressure deformation has plastic character (site BC) (fig. 2), and at value of the pressure σ_{LIM} there is destruction of a body.

Bones are characterized by very high mechanical strength. Depending on type of a bone and its size mechanical destruction begins after achievement of the pressure 10^7-$10^8 \frac{N}{m^2}$. In comparison with bones durability (strength) of other biomaterials is low for example: the tissue of walls of large arteries maintains only a pressure of $\approx 3 \cdot 10^6 N/m^2$.

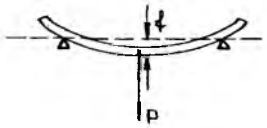


Fig.3

There are various methods of determination of a module of elasticity. We shall determine the module of elasticity at *bend deformation.*

If to the middle of the direct (straight) elastic tube freely laying on firm support force P is enclosed, the tube is bent (fig. 3). Moving f, which is received the middle of a tube is called the *sag.* Sag (or **bending deflection**) depends on load of the tube and its module of elasticity. For the sample of a bone in the form of a tube the theory gives the following expression for the module of elasticity:

$$E = \frac{P \cdot \ell^3}{12\pi f(R^4 - r^4)}, \qquad (3)$$

where P is load, ℓ is length of the sample (distance between supports), R and r are accordingly external and internal radius of the tube.

Description of installation

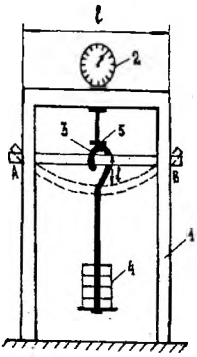


Fig. 4

Installation for determination of the module of elasticity of a bone (fig. 4) consists of the П-shaped support 1, on which the indicator of small displacements 2 (with the scale division value of 0.1 mm) for measurement of the sag f is fixed. In apertures A and B of the support is placed a sample (the bone or a duralumin tube), on which middle put on the collar 3 of core for hanging of loads 4. Weight of one load is 2 N. The core of the indicator place in the horizontal platform 5 in the top part of the collar so that the arrow of the indicator has made 2-3 full revolutions.

By rotation of the scale of the indicator the arrow establish on zero. Cautiously hanging loads, determine on the indicator the **sag f** of the sample (the red scale).

Order of the work

I. Study of dependence of the deformation on a load.

1. Prepare installation with a researched sample as it is described above.
2. Increasing the load from 0 up to 20 N by 2N write down in the table 1 sag f corresponding to each load for the bone and the duralumin tubes.

Table 1

	Bone					Duralumin						
P, N												
f, m												

3. Construct graphs f = f (P) for the bone and duralumin tubes. If for the given loads deformation was elastic, the graph should represent a straight line.

2. Determination of the module of elasticity of the bone and duralumin tubes.

1. Measure three times by vernier calipers in several places the external and internal diameter of the bone and find average values of external <R> and internal <r> radiuses of samples.
2. Measure internal and external diameters of the duralumin tube. Data write down into the table 2.

Table 2

R, m	<R>, m	r, m	<r>, m	ℓ, m	P, N	f, m	E, N/m^2
1. 2. 3.							

3. Substitute in the formula (3) one of the values of loading P and sag f corresponding to it and calculate E (N/m^2) for two samples, compare with tabulared values.

Conclusion: 1) at small loads deformation of the bone has elastic character (obeys the law of Hooke); 2) the module of elasticity of the duralumin is more than at the bone that is at identical sizes of samples and loads the bone has a smaller sag.

Control questions

1. What is a deformation? The basic kinds of deformations.
2. Define the absolute elongation, percent elongation, mechanical pressure.
3. Formulate Hooke`s law, draw the graph $\sigma = f(\varepsilon)$.
4. What is the Young`s modulus? What are its units? Physical sense of Young`s modulus.
5. How does determine the module of elasticity in the given work?
6. What are the scale division values of the vernier calipers and the indicator of small displacements?

LABORATORY WORK №11

SPECTRAL CHARACTERISTIC TEST OF EAR ON THRESHOLD OF AUDIBILITY

Purpose of the work: to determine limits of sound frequencies perceived by human ear, to construct the graph of frequency dependence of level of intensity on threshold of audibility.

Equipment: the sound generator (GS-33 or GS-34) with attenuator, headphones.

Theory of the work

Sound perceived by human ear is longitudinal mechanical waves in elastic mediums (solid, liquid, gaseous) of frequency 16 – 20000 Hz. The range of perceived frequencies is individual and can decrease with the years or as result of illnesses.

The sound wave creates acoustic pressure (superfluous above atmospheric pressure), transfers energy and is distributed in a medium with the speed v, dependent on elasticity E, density of medium ρ and temperature:

$$\upsilon = \sqrt{\frac{E}{\rho}}$$

Physical characteristics of sound are: 1) **frequency v**, 2) **spectral structure f(v)** (for a complex sound), 3) **intensity I** (or sound pressure) and corresponding to them *psychophysical* **characteristics**: 1) **height of tone**, 2) **timbre**, 3) **loudness**. It is possible to show that intensity I of a flat wave is connected with acoustic pressure P_m by dependence:

$$J = \frac{\Delta P_m^2}{2\rho\vartheta}, \qquad (1)$$

ρ is density of a medium, ϑ is speed of the wave, ΔP_m is peak value of pressure.

To bigger frequency of sound corresponds the bigger height of tone. Sound of the same frequency or « pure tone » seldom meets in the nature. More often we deal with the complex sounds described by complex periodic functions, which according *to theorem of Fourie* can be presented as the sum of harmonious oscillations. The sound with the *least* frequency is the *basic tone* and sounds with higher frequencies are the *overtones*, determining the timbre of the sound.

Intensity of sound is quantity of energy transferable per unit of time per unit of area perpendicular to sound wave. Intensity is measured in $\frac{Wt}{m^2}$.

Human ear of perceives sounds of intensity from $I_0 = 10^{-12}$ Wt/m^2 up to I_{max} = 10 Wt/m^2 for frequency of 1000 Hz. The first minimal value of intensity is *threshold of audibility*, the second maximal is *painful threshold*. Sound pressure corresponding to these thresholds: $P_0 = 2 \cdot 10^{-5} Pa$ and P_{max} = 60 Pa.

As the range of intensity of perceived sounds is very great ($\frac{I_{max}}{I_0} = 10^{13}$), it appears convenient to compare intensity of sounds in logarithmic scale. In this connection the value of **level of intensity L** of sound is entered:

$$L_B = \lg\frac{I}{I_0}, \qquad (1)$$

where I is intensity of researched sound; I_0 is intensity of sound on threshold of audibility.

The level of intensity is measured in this scale in Bell (B). From (1) follows that 1 Bell is unit of scale of levels of sound intensity corresponding to change of intensity in 10 times. Usually is applied unit in 10 times smaller, that is decibel (dB). In this case the formula (1):

$$L_{dB} = 10\lg\frac{I}{I_0}. \qquad (2)$$

Or $$L_{dB} = 20\lg\frac{P}{P_0}$$ for pressure p.

The level of intensity of sound characterizes a sound from the physical point of view. For estimation of subjective perception the concept of loudness is entered. **Loudness** is the value describing acoustical sensation of the given sound and dependent from its intensity and spectrum of frequencies. For example, ultrasounds and infrasounds are characterized by the certain intensity, but loudness of them is equal to zero, as these sounds are not perceived by human ear.

For measurement of loudness the concept of *level of loudness E,* which is measured *in phones* is entered. In the basis of creation of scale of levels of loudness **psychophysical law of Weber-Fehner** lays. According to this law: *if to increase irritation in geometrical progression (on identical value), for example, if intensity of a sound accepts number of consecutive values: aI_0; a^2I_0; a^3I_0 ($a > 1$ is some coefficient),than values of loudness of sound corresponding to them will be equal to E_0, $2E_0$, $3E_0$.* It means that **loudness of sound is directly proportional to logarithm of intensity**:

$$\boxed{E = kL} \quad \text{or} \quad \boxed{E = k\lg\frac{I}{I_0}}, \qquad (4)$$

where k is coefficient of proportionality dependent on intensity and frequency of the researched sound; k is equal to 1 for frequency of 1000 Hz.

To determine the level of loudness of any sound have agreed to take tone of frequency 1 kHz and to change its intensity until its loudness does become identical with loudness of determined sound. The level of intensity in decibels of this tone will be numerically equal to the level of loudness of determined sound in phones.

On the basis of many experiments with people curves of equal loudness, i.e. L = f (v), [I= f (v)] have been received at E = const (fig. 1). Each curve corresponds to the certain level of loudness in phones. The bottom curve corresponds to intensities with the weakest heard sounds *to threshold of audibility.* For all

Fig. 3

frequencies of this curve $E_{ph=0}$. The top curve corresponds *to threshold of painful sensation.*

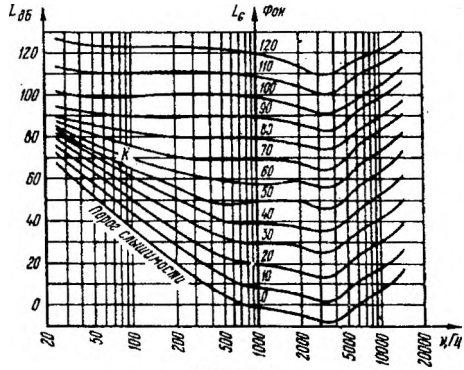


Fig.1

Method of measurement of sensory acuity is called **audiometry**. At audiometry on the special device *(audiometer)* can be determined threshold of acoustical sensation on different frequencies. The received curve is known *as spectral characteristic of ear on threshold of audibility.* On deviation of this curve from the normal curve it is possible to diagnose diseases of an ear.

Description of installation

Research of sensory acuity of a person is made with the help of the electro-acoustic device that is known as *audiometer* or the sound generator, representing the source of sine wave electric oscillations of sound and ultrasonic frequencies in the range of 20 – 200 000 Hz. Generator is supplied by attenuator; it is the device for regulation of target voltage (capacity) in the set number of times measured in dB. The basic adjustments located on the forward panel of the generator are shown in figure 2.

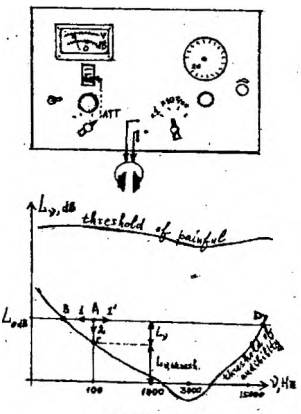


Fig.2

At performance of the work to reach a threshold of audibility is possible by two ways. Let, for example on frequency of 100 Hz on the scale of attenuator we have established 0 dB. Thus "Output" is 0.78 V that corresponds to the certain level of intensity of sound L_0 (the horizontal line in figure 2). In headphones thus you will hear pure tone of the certain loudness (point A).

For achievement of threshold of audibility is possible to move in the direction 1 to the point B (the bottom border of perceived frequencies). Thus, apparently from figure frequency decreases **and L=const**. If to increase frequency (direction 1`) we also shall achieve threshold of audibility in the point D (the top border of perceived frequencies). The second way of achievement of threshold of audibility is movement from the point A to the point C. Thus we reduce level of intensity L of sound and **frequency does not change (v=const).**

Procedure of the work

I. Determination of perceived sound frequencies (L=const).

1) Connect headphones to plugs " Output " and put on headphones;

2) establish on scale of attenuator 0 dB on frequency of 100 Hz;

3) for determination of the bottom border of perceived frequencies $\nu_{bot.}$ smoothly reduce frequency before disappearance of sound in headphones. Establish frequency a little bit less of ν_{bot} and smoothly increasing frequency repeatedly find the bottom border of perceived frequencies (occurrence of sound).

4) Measurements repeat some times and find average value of $\nu_{bot\ aver}$

5) Similarly find average value of the top border of perceived frequencies on a threshold of audibility $\nu_{top\ aver}$, increasing frequency from 10 000 Hz. Results bring into the table 1.

Table 1

Bottom boundary, ν_{bot}			Top boundary, ν_{top}		
ν_1	ν_2	$\nu_{bot\ aver}$	ν_1	ν_2	$\nu_{top\ aver.}$

Conclusion: the range of frequencies perceived by an ear of the person depends on sensory acuity and operating conditions at determination of frequencies.

2. Construction of graph of dependence of level of intensity from frequency on threshold of audibility (ν=const).

1. By the handle « ADJUSTMENT of OUTPUT » establish arrow of the voltmeter on zero of scale of dB. Establish frequency of 125 Hz.
2. By switches «LIMITS of SCALES. DECREASING» and « ADJUSTMENT of OUTPUT» receive disappearance of sound in headphones. The sum $L_\nu = L_1 + L_2$ specifies (fig. 3) on how many dB the target voltage is weakened in comparison with initial (equal to 0.78 V) for sound of frequency 125 Hz for achievement of threshold of audibility, where L_1 is indication of the attenuator, L_2 is indications of voltmeter in dB (taking into account of sign minus of L).
3. Similarly determine L_ν for other frequencies: 250, 500, 1000, 2000, 4000, 8000 Hz.

4.Calculate threshold level of intensity of sound ($L_{\nu\ thresh}$) for each frequency under the formula:

$$L_{\nu\ thresh} = L_\nu - L_{1000}, \qquad (5)$$

where L_{1000} is value of decreasing of voltage in dB for sound frequency of 1000 Hz; L_ν is value of decreasing of voltage in dB for sound of frequency ν.

5. All results bring into the table 2.

Construct the graph $L_{\nu\ thresh} = f(\lg\nu)$.

Table 2

ν, Hz / L_ν, dB	125	250	500	1000	2000	4000	8000
L_ν							
$L_{\nu thresh}=L_\nu-L_{1000}$							

Conclusion: sensitivity of ear to sounds of different frequencies is various. The maximal sensitivity of ear is observed in the range of 1000 – 3000 Hz. On low and high frequencies sensitivity of ear goes down.

Control questions

1. What is the range of frequencies perceived by human ear? What is the frequency interval of maximal ear sensitivity?
2. What is the intensity of sound? Units of measurement of intensity.
3. Physical characteristics of sound and physiological characteristics corresponding to them.
4. Define threshold of audibility and painful threshold with their values of intensity and sound pressure.
5. Dependence between intensity of sound and sound pressure.
6. What is the level of intensity of sound?
7. Psychophysical law of Weber-Fechner. Units of L and E.
8. The level of intensity of a sound on frequency of 200 Hz is equal to 50 dB. Determine loudness of this sound. Use the fig.1.

LABORATORY WORK №12

Study of action of ultrasonic oscillations on substance and determination of wavelength and speed of propagation of ultrasound

Purpose of the work: to study some features of propagation of ultrasound in the liquid medium, to learn to determine wavelength and speed of propagation of ultrasound.

Equipment: the device for ultrasonic therapy UST-101, cylindrical vessel with a focusing bottom, a vessel with plane-parallel walls and flat bottom, source of light , support, thermometer with the scale division value of 0.5 ^{0}C, starch, vaseline.

Theory of the work

Ultrasound is elastic vibrations and waves, which frequency occupies a range from 20 kHz (the bottom border of the frequencies perceived by the person with normal hearing) up to 10^{10} Hz.

In medicine US is received with help: a) of the phenomenon of the converse piezoeffect and b) the phenomenon of magnetostriction.

Converse piezoeffect is the change of thickness of a plate which has been cut out from some crystals (quartz, lead zirconate titanate, synthetic ceramic etc.) under action of the alternating voltage enclosed to sides of the plate.

For quartz, for example, are characteristic the structural groupings, containing three atoms of silicon and six atoms of oxygen symmetric concerning of propagation of charges. At deformation the grouping is done polar. Thus on surfaces of the crystal there are electric charges which mark depends on character of deformation. This phenomenon is called **direct piezoeffect**. It is used for generation of US.

If on the end faces of the plate of quartz to put the alternating electric voltage, thickness of plate will increase and decrease according to frequency of alternating current that will cause underpressure and a condensation of particles of medium with occurrence of US wave. It will be **converse piezoeffect** of frequency 200 kHz – 50 MHz. The **ultrasonic generator** of US consists of the source of alternative voltage (1), an oscillatory contour LC (2) and piezoplate (3) to which the voltage from the oscillatory contour is brought (fig. 2). When frequency of the voltage brought to the plate from the oscillatory contour will be equal to own frequency of mechanical fluctuations of the plate, i.e. $\nu_{plate} = \nu_{gen}$, we'll have a resonance. Thus the amplitude of oscillations of the plate and energy of ultrasound will be **maximal**.

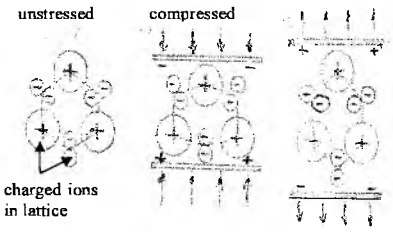


Fig. 1

The effect of **magnetostriction**: if to place a core from ferromagnetic material (alloys of Fe+Ni) in the variable high-frequency magnetic field directed along it, the length of the core will be change and its end faces will radiate low-frequency (0-200 kHz) ultrasound.

Propagation of ultrasonic waves submits to laws general for all acoustic waves (laws of reflection, refraction, dispersion). But the ultrasound has also some **features** of propagation: 1) the ultrasound has smaller wavelength, than sound in air. It allows focus ultrasonic oscillations easily. Because of small length of wave US is distributed as narrow beam. 2) Speed of propagation of US strongly depends on properties of a medium: in solid bodies its speed is much more than in liquids and air. Ultrasound is absorbed in air approximately in 1000 times more, than in

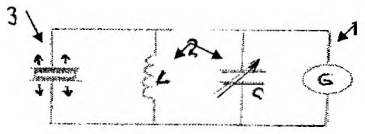


Fig. 2

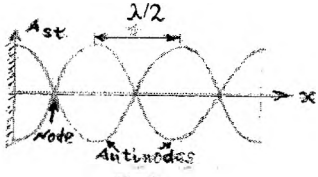


Fig 3

water. Therefore contact between a radiator and object should not contain an air layer. As contact substance which is rendered on a radiator is used water, oil, gel, vaseline, etc.

Wave properties of ultrasound. Standing wave. Reflection and refraction of

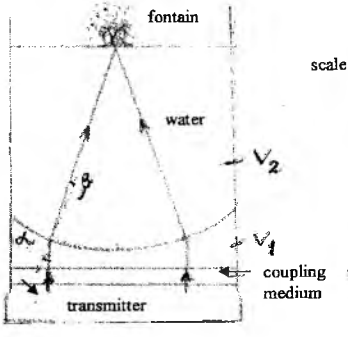


Fig. 4

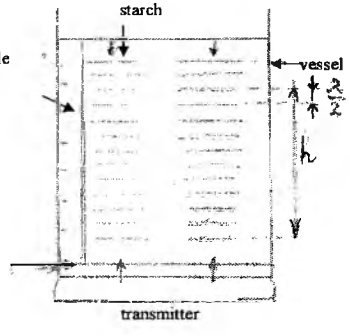


Fig. 5

ultrasound is occurred under laws of geometrical optics. Therefore we can focus ultrasound with help of glass filled with water and glass bottom as the concave lens (fig. 3). US wave refracts on the border "glass - water" and the Snell refraction law will be noted so: $\frac{\sin\alpha}{\sin\beta} = \frac{V_1}{V_2}$, where α is incident angle, β is angle of refraction, v_1

is the speed of US in the 1-st medium (glass), v_2 is speed of US in the 2-nd medium (water).

As speed of ultrasound in glass v_1 is more than speed in water v_2, also the incident angle α will be more then angle of refraction β, i.e. beams will come in the same point. In the point of focus there is a substantial increase of temperature and crushing of water on fine particles (aerosol). The focused ultrasound is used in surgery for section of tissues, in physiotherapy for manufacturing of aerosols of medicinal substances, in pharmacy for reception of emulsions and suspensions.

Special case of interference is **standing waves**. They are formed at addition of two waves: 1) from transmitter and 2) reflected from border of two mediums (fig 5). For example, if one end of a long cord to fix motionlessly and other end to result in oscillatory movement, that wave reflected in the point of fastening will be interferate with the ingoing wave and forms a standing wave.

Points of a standing wave for which the amplitude of oscillations is maximal are named **antinodes**, and points in which amplitude of oscillations are minimal are known as **nodes** of the standing wave. Distance between the next nodes is equall $\lambda/2$ (fig 3). Long existence of a standing wave (mechanical or electromagnetic) in biological tissues can results to its **local overheat** (in antinodes) and at presence of US standing wave in blood vessels results to concentration of erythrocytes in the antinodes, that decreases supply of tissues by oxygen.

Actions of ultrasound:

Thermal action. It is caused by absorption of energy of ultrasound. The thermal effect causes expansion of tissues, blood vessels and as consequence, strengthening of a blood-groove. Due to thermal effect the focused ultrasound can be used as scalpel for soft and bone tissues.

Mechanical action. Fluctuation of pressure in ultrasonic field causes micromassage of tissues, there is a microvibration on cellular and subcellular levels, destruction of biomakromolecules, microorganisms, viruses, malignant tumours and stones in kidneys. The ultrasound causes damages and reorganization of cellular membranes, changes its permeability. At propagation of ultrasound in a medium there is a variable sound pressure, that accepts positive value in the field of compression (+3atm) and negative in the area following it (-3atm). It results to formation of **breaks** of continuous liquid with formation of microscopic cavities (**cavitation**). When on the place of a cavity the site of compression is formed, it slams quickly, allocated significant quantity of energy in small volume that results to destruction of microstructures of substance.

Physical and chemical action. The ultrasound causes acceleration of diffusion, ultrasonic luminescence, formation of a potential difference in biological tissues, acceleration of some chemical reactions.

Medical and biologic application of ultrasound can be divided on some directions: diagnostics, therapy and surgery. To the first direction concerns location methods with using of pulse radiation. It is detection of tumours in soft tissues and cracks in

bones, determination of tumors and edema of a brain (echoencephalography). Location methods are based on reflection of ultrasound from the border of two mediums with different density. To this method concerns ultrasonic cardiography also, i. e. measurement of the sizes of heart in dynamics; determination of the sizes of eye mediums in ophthalmology. Ultrasonic effect of Doppler is used for study of character of movement of mitral valves and speed of a blood-groove.

Ultrasonic therapy concerns to the second direction with use of ultrasound of frequency 800 kHz and intensity of 1 Wt/cm^2 and less. Initial mechanisms of action are mechanical and thermal actions.

Description of installation

The device for ultrasonic therapy UST-101 is intended for generating of ultrasonic oscillations. It works in pulse and continuous modes of generation from the network of alternating current with the voltage of 220 V. Maximal capacity of ultrasonic oscillations makes 4 Wt ± 40 %.

On the lobby panel of the device are located: procedural timer; the step switch « INTENSITY, Wt/cm^2 » with steps:1,0; 0,7; 0,4; 0,2; 0,05 Wt/cm^2; the switch "network"; the indicator of a target voltage; the indicator of network; a socket for connection of a cable of the radiator; switch " Radiators ", switch " Operating mode ".

Steps of the work

1. Preparation of the device for work.

1. Press button " Radiators " №4.
2. Establish intensity of 1 Wt/cm^2 having pressed the corresponding button.
3. Establish continuous operating mode (button " 1 ").
4. Press the key "network" and turn the handle of the timer against the stop clockwise.

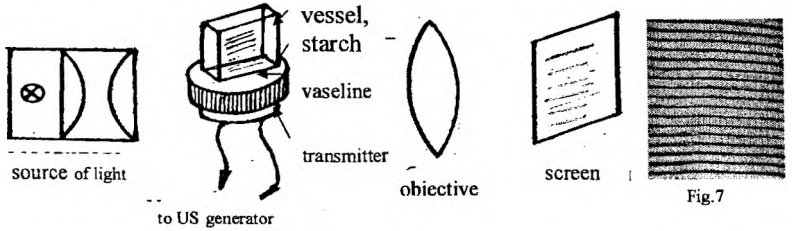


Fig.6 Fig.7

2. Determination of the wavelength and speed of ultrasound in water.

1. Collect the installation (fig. 6).
2. In the vessel with plane-parallel walls and flat bottom pour a solution of starch in water, stir it and switch-on the generator. Ultrasonic oscillations will be reflected from the surface of liquid in the vessel and will form a standing

ultrasonic wave, in the antinodes of standing wave will concentrates particles of starch forming parallel strips (fig. 7), distance between strips is ½λ.

3. Measure the greatest possible number of distances (n) on height of the vessel h, and from the formula $\frac{\lambda}{2}n = h$ determine wavelength of US: $\lambda = \frac{2h}{n}$ (m).
4. For calculation of speed of ultrasound in water $\vartheta = \lambda\nu$ take the frequency of the generator 880000 Hz.
5. Put all dates in the table 1:

Table 1

n_1	n_2	n_3	n_{aver}	h, m	λ, m	ν, Hz	v, m/s
						$88 \cdot 10^4$	

3. Focussing of ultrasound.

1. Establish intensity of 1 Wt/cm^2 and place the cylindrical vessel with the focusing bottom (bottom of the vessel is a concave lens) greased vaseline.
2. Pour water (or a spirit) up to the label corresponding to the focal length of the vessel, swith-on the source of light and the device.
3. To pay attention to the received effect (fountain and aerosol).

4. Heating of substance by ultrasound and determination of useful capacity of the generator.

1. In the point of the vessel with a focusing bottom in which there was a fountain to lower the thermometer and to mark its indications each 30 seconds within 3 minutes.
2. Given to bring in the table 2.
3. Determine useful capacity of the generator under the formula: $N = \frac{Q}{\tau} = \frac{cm(t_2 - t_1)}{\tau}$, where m is mass of water in the vessel (m=23 g), $(t_2 - t_1)$ is change of temperature in time τ, c is specific thermal capacity of water ($c \approx 4200$ J/(kg · K).
4. Construct the graph of dependence of temperature on time $t=f(\tau)$.

Table 2

τ, s	0	30	60	90	120	150	m, kg	c, J/kg·K	t_1, s	t_2, s	τ, s	Q, J	N, Wt
t,^{0}C							0.023	4200					

5. Supervision of interference and diffraction of ultrasonic waves.

Place on the radiator of US the flat vessel with two reflectors 1 (fig. 8). Pour a solution of starch, stir it, switch on the light and device UST and observe the specified phenomena (fig. 8, 9).

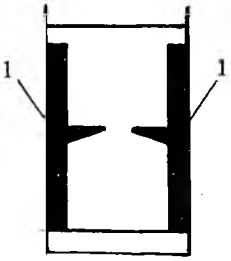


Fig. 8

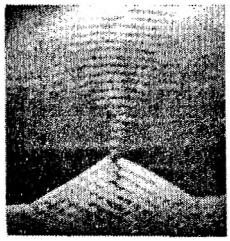


Fig. 9

Control questions

1. What are the ultrasonic oscillations?
2. Explain reception of ultrasound with the help of converse piezoelectric effect. What is magnetostriction?
3. What are the basic features of propagation of ultrasound in a medium?
4. What is the condition of the maximal amplitude of oscillations of piezoelement?
5. What is the condition of reflection of ultrasound from the border of two mediums?
6. How does focus ultrasound?
7. What is cavitation in the liquid medium?
8. Specify the basic mechanisms of action of ultrasound on substance.
9. What are the basic applications of ultrasound in medicine?
10. Working formulas for determination of λ and v.

Laboratory work №13
Biophysical bases of measurement of arterial pressure

Purpose of the work: study of biophysical bases of measurement of arterial pressure (AP or BP, that is blood pressure); correct measurement of AP by the most wide-spread methods, including Korotkov`s method; to master a procedure of comparison of parameters of two samples by the method of check of statistical hypotheses.

Equipment: tonometers, phonendoscope for measurement of AP; statistical tables.

Theory of the work

Control of AP underlies of diagnostics of many diseases, including such wide-spread as arterial hypertension that meets at ≈40% of adult population. According to opinion of WHO experts normal systolic AP should be < 140 mm Hg and diastolic should be less than 90 mm Hg *irrespective of age and a gender* (classification of levels AP see in the table 1).

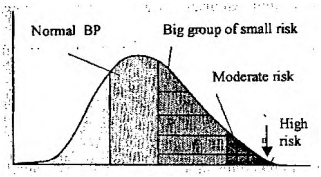


Fig. 1

Value of BP (systolic and diastolic) in a big population is distributed under the normal law (fig. 1), and the population majority keeps within a group of normal BP. Persons with very high BP (black colour on fig. 1) are subjected to the big risk of the insult and ischemic heart disease, but number of such persons in all population is small. Therefore, even if all of them would receive the most effective treatment, frequency of insults and sharp heart attacks in all population would decrease inappreciably (because the majority of cardiovascular complications happens at people with normal or small increase of BP). Hence, BP decrease in all population only on some mm Hg (that can be attained conducting of socially-hygienic activities) will allow to reduce common sickness rate by cardiovascular diseases considerably in a greater degree in comparison only with larger decrease of BP at small number of persons of group of high risk. So, shift of bell-shaped curve of BP distribution in population to the left on 5 mm Hg would lead to decrease of the sickness rate by insult on 40 % and ischemic heart trouble on 20%.

Table 1

Categories of BP	Sistolic Pressure	Diastolic Pressure
Normal	Less than 130	Less than 85
Increased normal	130-139	85-89
	Arterial hypertension	
Stage I	140-159	90-99
Stage II	160-179	100-109
Hypertensive crisis	More than 180	More than 110

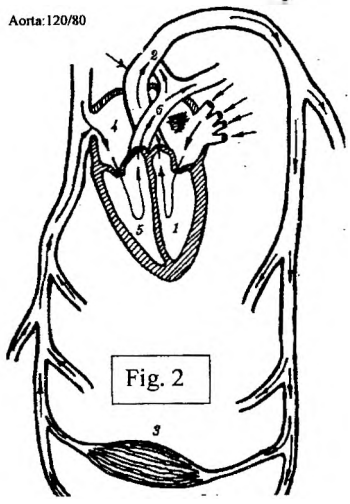


Fig. 2

The level of a blood pressure is defined and supported due to interaction of two factors: 1) neurohumoral (control action of brain on force, pulse rate and diameter of blood vessels), and 2) hemodynamic (volume of blood in vascular system, vascular resistance, viscosity of blood) immediately defining value of AP.

Let's consider some laws of hydrodynamics as base of hemodynamic factor of AP.

The left and right halves of heart (each half consists of the auricle and ventricle connected among them by valves, fig. 2) connects among them through the blood vessels, providing the system of tubes connected in series and parallel and providing blood flow continuity.

Heart is contracted as a unit. The phase of contraction *(systole)* lasts approximately 0.3 s. The left ventricle 1 contracts and creates overpressure above atmospheric and throws out arterial blood into the aorta 2. From aorta the blood through capillars 3 hits in all organs with homing of venous blood during *the diastole* (relaxation phenomenon) into the right auricle 4 (greater circulation). Simultaneously the right ventricle 5 being contracted throws out blood into the pulmonary arteria 6 under the pressure in 5 times less than in aorta, i.e. 120/5=24 mm Hg, forming the lesser circulation. Resistance of the lesser circulation is less therefore there is enough pressure in 24 mm Hg.

The diastole gives decreasing of overpressure in auricles and veins up to 0 mm Hg, that is up to atmospheric. As a result the greater circulation works due to the difference of pressures 120-0=120 mm Hg and lesser circulation due to difference of pressures 24-0=24 mm Hg, that makes 20 % from power of the greater circulation.

In the aorta pressure at a diastole falls not up to null, and up to 80 mm Hg, that provides the continuous prolongation of blood motion on the greater circulation.

So, the motion of blood on vessels is stipulated by presence of difference of pressures in the beginning and at the end of vessels. **The principal cause** creating this difference of pressures in vessels is a heart work. Other reason of a blood motion on vessels is contraction of skeletal muscles (it is said that muscles is «the second heart »), that gives compression of veins and due to presence in them of valves motion of blood is observed preferentially to the certain direction (aside heart). Besides, inflow of blood to heart on veins is promoted by the negative pressure in pleural area.

The work of heart in basic is stipulated by the left ventricle (work of the right ventricle is ≈0.2 of the work of the left ventricle). The work of the left ventricle is equal to sum of work $V_{,}p$ on injection under a pressure of blood into the aorta

(the static component) and work giving to blood a kinetic energy $mv^2/2$ (the kinetic component):

$$A_{left}=V_y p + mv^2/2 \qquad (1)$$

where V_y=60 ml is stroke volume of blood, pushed out by the ventricle, v = 0.5 m/s is velocity of blood in the aorta, p = 16 kPa is average pressure in aorta. As $m=\rho V_y$, where ρ=1050 kg/m^3 is density of blood, work of the left ventricle is A_{left} = 0.81 J. Taking into acount the right ventricle **work of all heart is** $A_h = A_{left} \cdot 1.2$ = 1 J.

The vascular system consists of parallel and in series connected tubes for which **the condition of a continuity of jet** satisfies: $s \cdot v$ = const, whence follows, that velocity of blood in a vessel of a different section is inversely proportional the areas of these sections. So, the sectional area of aorta in 600-800 times is less than the total sectional area of capillars: it means in 600-800 times velocity of blood in the aorta (0.5 m/s) will be more, than velocity in capillars (0.0003-0.0005 m/s).

For ideal liquid the total energy of some volume V at flowing on a tube remains constant. The total energy develops of a potential energy of pressure pV (it is created by an exterior source, the pompe), potential mgh and the kinetic $mv^2/2$ energies:

$$pV + mgh + mv^2/2 = \text{const} \qquad (2)$$

Having divided the right and left part of the equation (2) on V, we shall receive **a Bernoulli's equation**:

$$p + \rho gh + \rho v^2/2 = \text{const} \qquad (3)$$

The equation is well fulfilled also for real liquids with insignificant interior friction. Quantity p in (3) is known as static pressure, ρgh is hydrostatic pressure, $\rho v^2/2$ is dynamic pressure. The kinetic component $mv^2/2$ of heart work makes some percents from the value of total work of heart, because vascular system has the considerable resistance and about 60-80 % resistance of vascular system falls on arterioles and capillars.

Pressure difference in a vessel can be found from equation of **Hagen - Poiseuille**:

$$p_0 - p = \Delta p = Q \cdot Z, \qquad (4)$$

where $\Delta p = p_0 - p$ is difference of pressures in the beginning (p_0) and in the end (p) of the vessel, $Z=8\eta l / \pi r^4$ is hydraulic resistance of the vessel of radius r and length l, Q is volumetric flow rate of blood (the volume, which is flowing per 1 s through 1 m^2 of a sectional area). It is possible to find Q from the Poiseuille formula: $Q = \Delta p \cdot \pi \cdot r^4/8\eta l$. Then, knowing the volumetric flow rate Q and value of the vascular resistance is possible from (4) to find the pressure of blood p in any point of vascular system:

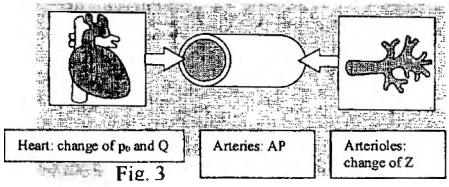


Fig. 3

$$p = p_0 - Q \cdot Z, \qquad (5)$$

where p_0 is blood pressure in the ventricle, Z is vascular resistance between the ventricle and the given point. From (5) **the important conclusion** follows: **value of pressure**

in any point of vascular system can be controlled by the way of:
1) change of initial pressure p_0 in the heart, 2) change of volumetric flow rate Q, 3) change of resistance of vessels Z. Changes of p_0 and Q can happen as result of change of operating mode of heart and change of Z can happen due to change of a diameters of vessels (not large arteries and capillars, and the fine arterioles, which walls contain smooth muscles cells, capable to be contract: see fig. 3).
Blood flow in vascular system in norm has laminar character. At physical activity or in the places of sharp vasoconstriction flow becomes *turbulent* being accompanied by sound appearance.
For measurement of arterial pressure we can use direct and indirect methods (see tab. 2): method of palpation (palpatatory), auscultatory, oscillometry, etc.
We shall consider the most widespread of them.

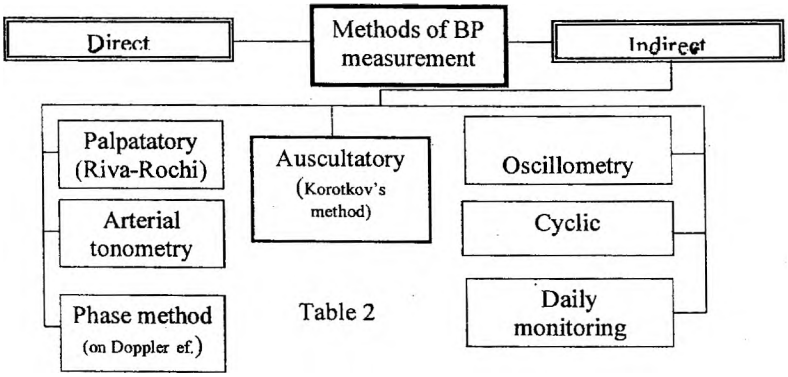


Table 2

Direct methods.
Most exact method of AP measurement is the **direct endarterial method** and the standard of AP quantity is the pressure of blood in the aorta. In surgical practice direct measurement of pressure in the cardial cavity is created by a method of catheterization (fig. 4), i.e. by introduction through one of large vessels of a probe on which end there is a small-type electromanometer of diameter 1-2 mm. The data unit of the probe is silicone resistance, connected with membrane accepting exterior pressure. The catheterization of blood vessels for the first time has been carried out in 40-s of 20 century and since it is one of the important methods of deriving of the information on a cardiac performance.

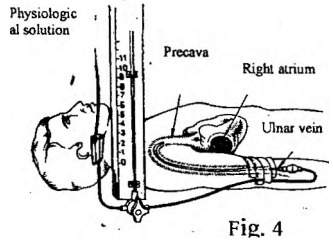


Fig. 4

For the first time pressure has been measured by English priest A. Hales in 1733: he has connected a flexible tube the femoral arteria of the horse to a long vertical brass tube with the unclosed upper end. After declamping on the connecting tube, the blood has filled in the brass tube and has risen up to height of 2 m. Pressure of the blood column in the tube was counterbalanced by arterial pressure and was ≈20

kPa (human AP is 16 kPa). The level of blood in the tube changed with frequency of heartbeats. Later, in 1834 Poiseuille for decreasing of height of raising blood has put the mercury manometer to brass tube (therefore until now unit of pressure is mm Hg), and Ludwig having added a float has devised the kymograph (Gk. Kyma=wave; grapheon=stylos), which has allowed to provide the continuous record of blood pressure.

Indirect methods.

The bloodless method of BP measurement was offered in 1896 year by the Italian doctor **S. Riva-Rocci**. The method provides using of the sphygmomanometer (from Gr. sphygmos = impulse) and a cuff with mercury manometer for measuring pressure in it (fig. 5). His **palpation method** allows to determine with particular precision **only the systolic pressure**. As well as in Korotkov's method, in the cuff superimposed on the shoulder (or on the femur) is created the pressure exceeding a maximal pressure in the radial arteria on 20-30 mm Hg, thus a pulsation stops. Having slightly opened the valve, slowly release air from the cuff marking *by palpation* on the field of arteria below cuff indications of the manometer at the moment of occurrence of sphygmus that corresponds to a systolic pressure. At further pressure decrease pulse becomes more and more clear, but at the particular moment its boosted intensity disappears and it gains usual properties. At the moment of last strong pulse wave the manometer will show value of a diastolic pressure. However, diastolic pressure to find by this method is more difficult, and systolic pressure as rule on 5-15 mm Hg is less, than at auscultatotary, therefore Korotkov method is used everywhere. Korotkov (1874-1920) has altered method of Riva-Rocci so, that it was possible to measure precisely a diastolic pressure too.

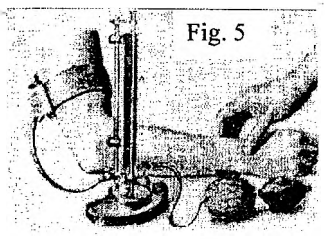


Fig. 5

Let's consider biophysical bases of Korotkov's method (fig. 6).

1). Between the shoulder and elbow on the arm 2 is superimposed the cuff 1 at a

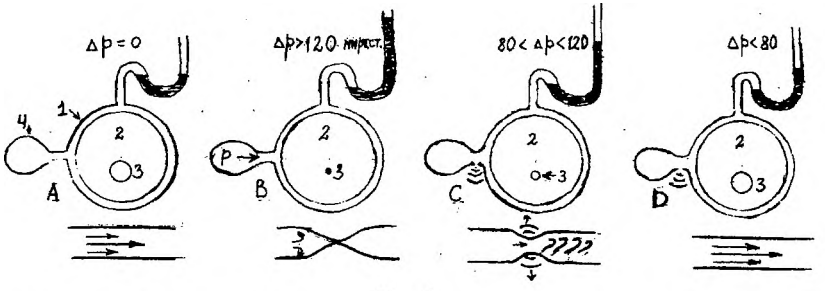


Fig. 6

level of heart, in this case pressure in the arteria 3 will coincide with pressure of blood in the proximal to heart part of aorta. Thus the overpressure Δp of air in the cuff misses (A).

2). At injecting air into the cuff with help of the pump 4 is created the pressure squeezing the humeral arteria. When pressure in the cuff will exceed systolic ($\Delta p >$ 120 mm Hg in norm) blood flow stops (B). If the arm is weakened, pressure of air inside the cuff approximately equal to pressure in the soft tissues adjoining with the cuff (the **basic physical idea** of the method).

3). Releasing air from the cuff with the help of the valve, we reduce pressure in the soft tissues also (C). When the pressure in the arteria becomes equal to systolic, the blood will start to be forced through arteria. It is accompanied by sounds (**Korotkov`s sounds**) listened in the phonendoscope imposed on the humeral arteria.

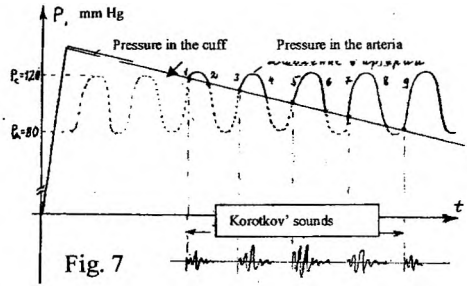


Fig. 7

Korotkov' sounds are stipulated: a) by vibration of walls of the arteria under activity of impacts from the portions of blood, that are forced through the squeezed field of the vessel that is accompanied by formation of a weak shock wave, b) on these sounds are imposed sounds caused by a turbulence of blood flow transiting through narrow spot of the vessel. On the fig. 7 straight line shows change of pressure in the cuff, and change of pressure in the arteria is shown as a wave. On the parts 1-2, 3-4, 5-6, 7-8 pressure in arteria is more than in the cuff, therefore in the points 1,3,5,7,9, where pressure in the cuff and arterias is equalized, the arteria sharply straightens, giving in oscillations environmental tissues with formation of Korotkov` sounds. There are 5 phases of Korotkov' sounds (see tab. 3).

4). When the pressure in the cuff becomes less of diastolic (the point 9 on fig. 7), the arteria finally straightens, interruption of the blood flow is stopped, sounds disappear. Disappearance of last tone corresponds to the diastolic pressure.

Table 3

I	Occurrence of faint sound which in process of air outlet from the cuff become more accurate and intensive. First from at least 2 consecutive sounds corresponds to a **systolic AP**.
II	Sounds are associated to tones with rustling tint.
III	Tones become more accurate and intensive, with a crackling tint.
IV	Acute decreasing of tones which becomes deaf and blowing.
V	Disappearance of last tone that corresponds to a **diastolic** AP.

The oscillometric method offered in 1876 by E. Maray intends location of a human finiteness in the special device (a water plethysmograph), creating adjustable pressure on finiteness and simultaneously registering small pulsings of volume of finiteness. Later from a plethysmograph have refused in favour of the usual cuff, that have united the device for making of pressure and the data unit of pulsings of artery. At air exhaust from the

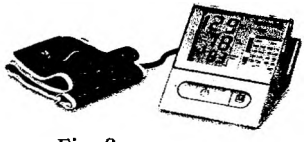


Fig. 8

cuff pressure in the cuff for which pulsings starts to grow sharply, corresponds to the systolic AP. When pulsings sharply decreases, it is corresponds to the diastolic pressure. Pulse changes of blood pressure in arteries under the cuff lead to small oscillations of pressure in the same cuff that is reflected in oscillations of level of mercury in anmanometer. In 1976 the first automatic blood pressure measuring device grounded on the oscillometric method has been released: pressure drop in the cuff falls by degrees on 6-8 mm Hg and at each step is analysed the amplitude of micropulsings of pressure in the cuff from arteries. Now blood pressure measuring devices on the basis of the *oscillometric method forms* ≈ *80 %* of all automatic and automanual devices (see fig.8) for measurement AP (including for measurement AP with the cuff on the wrist).

Cyclic methods of BP measuring are the most exact, but the measuring time is 2-3 minutes. The method is grounded on the continuous estimation of volume of arterial vessels of a finger by a photoplethysmography method (fig. 9). In the cuff wreathing the finger by means of electropneumatic system ЭПС controllabled by photosensor Ф is supported the pressure, that is reacting against a tension of arterial vessels transiting under the cuff. Hence, at maintenance of the constance of diameter of the finger arteries is sustained maintenance of close to null stretching pressure in arteries also. In this case pressure in the vessel iterates blood pressure in the finger arteries. As result the device displays the long-term time all diagramme BP by non-invasive method.

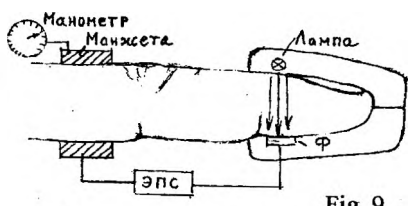


Fig. 9

Definition of BP **by a tonometry method** is featured for the first time in 1963 and guesses partial squeezing of superficially lying down arteries of finiteness (for example, on the wrist) and recording by means of built in the occlusive bracelet of strain-gauge indicators of the side pressure transmitted to them through a vascular wall. At the base of work of *strain-gauge indicator* lies the phenomenon of a tension resistive effect, consisting in change of resistance of conductors (for example, thin wire pasted on a paper) at their mechanical strain.

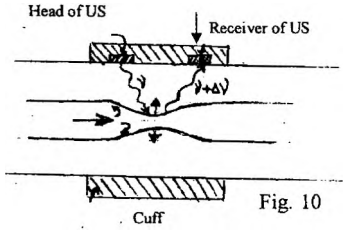


Fig. 10

The idea of study of a motion of arterial walls by means of ultrasonic sound (US) belongs to Vea and is carried out by Cardon (1971) in **a phasemetric method**. Under the cuff on a finiteness surface is imposed the ultrasonic head and the receiver of US, see fig. 10. The moving arterial wall reflects US and the reflected signal will be another frequency concerning the initial signal (Doppler effect):$\Delta\nu = 2\nu\, v/c$, where $\Delta\nu$ is the frequency shift of a beat lying in the heard frequency band, ν is frequency of the generator (ν = 8 MHz), v is velocity of

motion of the wall of artery, c is sound velocity in a tissue. As soon as pressure in the cuff becomes less systolic (see the point 1 on fig. 7) is heard a short click of openening of the artery and then click of closing of the artery (point 2 on fig. 7). At the further decrease of pressure, the time interval between both clicks becomes more and more while closing click will coincide with opening click. Their coincidence means that pressure in the cuff equal to diastolic. As Doppler is sensitive, this technique is usually used for measurement of low pressure, e. g. vascular insufficiency.

Long-term monitoring of BP.

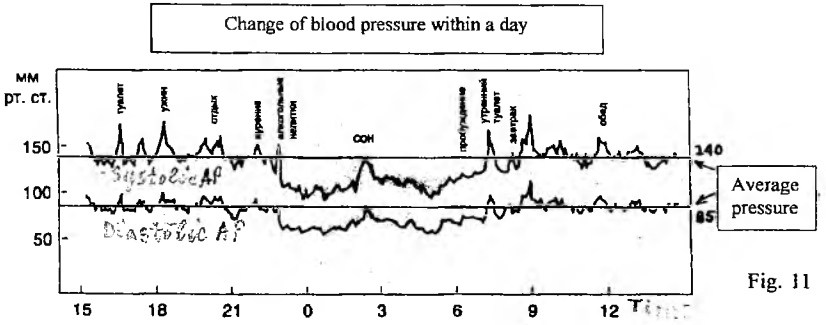


Fig. 11

BP is not stable even at a healthy person and depends on a time, a physical and mental loading, an emotional state (fig. 11) and many other factors. For example, the certain part of patients at visitation of the doctor has a phenomenon of "white smock": BP increases on 30-40 mm Hg in comparison with houme measurings. Therefore **the long-term monitoring of BP** (for example, within a day in each 0.5 hour) allows to find legitimacies of daily oscillations and opens additional diagnostic and medical possibilities.

Procedural bias (errors) of the cuff methods of BP measurement.

Let's consider errors of the most widespread method of BP measurement that is Korotkov's method.

a) At BP measurement by means of the cuff method in the end of a diastole takes place the instantaneous closing of the vessel that generates so-called *water hammer (or hydraulic (hemodynamic) shock)*. Hydraulic (hemodynamic) shock originates at unexpected closing of the fluid (blood) flow, streaming on a tube or at the instantaneous elimination of the flow closing. Pressure increasing at hemodynamic shock is ≈ 5-20 mm Hg.

b) The force field created by pressure in the cuff depends on *anatomical and physiological features of the arm*, therefore the errors in transmission of pressure from the cuff to the artery can have value of 4-12 mm Hg. Therefore at a considerable part of peoples BP on the *right arm is more* BP on the left arm.

c) From the same reason of the correct transmission of pressure from the cuff through a tissue to the artery, the cuff breadth should ≈ 20 % of diameter of

finiteness. If the patient is forced to hold of cuff clip with the free arm (owing to adiposity, at sportsmen, etc.), means, this cuff should be replaced.

d) At fast deflation of air from the cuff the given method tends to decrease systolic and to increase diastolic pressure (fig. 12). By straight line is shown change of pressure in the cuff against a wave of change of BP. The second, more "abrupt" straight line (fig. 12) corresponds to larger velocity of decompression. Systolic ("upper") and diastolic ("lower") pressures scored under indications of manometer are shown by fat points. It is visible from the fig. 12, that at fast decompression (fig. 12,б) errors will grow. In any case, it is necessary to lift promptly pressure to the level exceeding systolic and then to make slow **linear** decompression (if decompression decreases for example on exponent, because of sharp deceleration of pressure drop, it is difficult to find the moment of the termination of audibility of of Korotkov sounds).

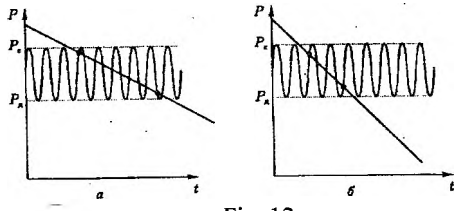


Fig. 12

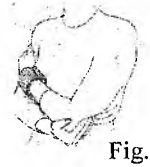


Fig.13

e) Position of the cuff concerning heart level influences manometer indications: if the cuff middle is placed above level of the right auricle on h metres, value of a BP decreases for value of hydrostatic pressure ρgh, ρ is blood density. If the cuff below heart level on h metres (for example, cuff is on a hip) pressure will be increased by quantity ρgh. Therefore in devices with the cuff on the wrist the cuff should be up to standard of heart (fig. 13).

Instrumental error. For the control over accuracy of BP metres there are national and international standards, there are most recognised: test report BHS (British society of hypertensia, 1993), and test report AAMI/ANSI (American association for improvement of medical apparatus, 1992). WHO experts recommend to use only the apparatus which have passed test under given reports in leading medical institutions. According to demands of these reports clinical trials are spent by two independent skilled experts on specially selected bunch of patients of different ages with different level of BP. For each patient with using certificated *mercury* (fig. 14)

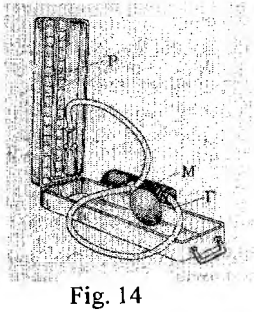


Fig. 14

sphygmomanometers («the gold standard») is carried out by Korotkov's method a series of consecutive control measurings and measurings by the tested device.To estimate accuracy of tested devices observed data compare to control results. Under report AAMI the mean of differences quantities of BP for the tested device and the expert should not exceed 5 mm Hg and a standard deviation should not exceed S=8 mm Hg. Measure of conformity to report BHS is appropriated accuracy rating (not less B/B accordingly for SAP and DAP). According to the table of observable differences between the given device and expert tonometer (report BHS) to the apparatus is appropriated accuracy rating from A (excellent) to D (unsatisfactory). For example, the highest accuracy rating A/A means, that not less than in 60 % of all measurings its indications differ from indications of a calibration instrument less than on 5 mm Hg (or the same, not less, than in 85 % of measurings an instrument reading differ from indications of a calibration instrument less, than on 10 mm Hg, in 95 % - on 15 mm Hg).

Grade of aqquracy	Percent of conformity of measurings on the examinee device in relation to measurings on the calibration instrument (the mercury standard, %)		
	≤ 5 mm Hg	≤ 10 mm Hg	≤ 15 mm Hg
A	60	85	95
B	50	75	90
C	40	65	85
D	< 40	< 65	< 85

For reception of data corresponding to the true BP on Korotkov's method, it is necessary to meet following

conditions and rules.

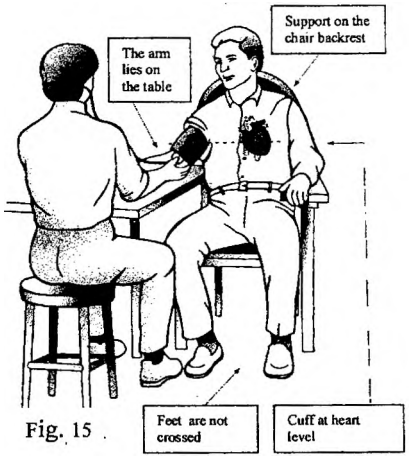


Fig. 15 .

1) *Measuring conditions.* 1 hour before to BP measurement is not recommended food intake (including tea, coffee), active physical and emotional loading, smoking.
2) *Position of a patient* (fig. 15): in a sitting position, the back leans against a chair backrest, the arm is had freely on a table (at elderly, diabetics, measurings make as well in a prone position and standing).
3) *Select of the arm of a patient.* At the first visitation of the doctor, BP measurement should be made on both arms. In norm the BP difference on the left and right arm is 5-10 mm Hg. More higher difference can be caused anatomical features or

a pathology of the humeral artery of the right or left arm. In the presence of a BP difference on arms more than 10/5 mm Hg (SAP/DAP) measurings make on an arm with higher indications of BP, and in the absence of the specified difference BP is measured on a non-working arm.

4) *Cuff position.* The cuff middle should be at heart level. If the cuff is placed below heart level than the BP is increased, if the cuff above this level BP is decreased. The cuff bottom edge should be on 2-2.5 sm above an ulnar fossa, between the cuff and a shoulder surface should transit a dactyl. The cuff is imposed on the unclothed arm, since at measuring through clothes indication are increased.

Table 5	
Cuff size	Size, см
Small	12x18
Standard	12x26
Large	12x40

5) *Cuff size.* The cuff should sweep not less than 80 % of round the arm (if round of the arm is more than 32 sm use the cuff of the incremented size). Usually standard cuff has the interior chamber in breadth 12-15 sm. For narrow cuff indications of measurement are increased, for wide cuff idications are decreased. The recommended sizes of cuffs are specified in the table 5 (BHS, 1997).
6) *Speed of inflation and deflation of air from a cuff.* Air inflation of air into a cuff should be fast (30 mm Hg /s), and deflation should be slow – 2-3 mm Hg/s.
7) *Phonendoscope position.* In the beginning it is necessary by palpution to find a place of the peak pulsing of the humeral artery and then to set a phonendoscope without squeezing in this place.
8) *Determination of SAP and DAP.* The moment of occurrence first of at least two consecutive tones is defined as the systolic BP. Disappearance of last distinct tone (V phase) corresponds to the diastolic BP. At some diseases was possiblly absence of V phase (phenomenon of the perpetual tone), in this situation for DAP accept the beginning of IV phase (sharp muting of tones).
9) *Number of measurements and intervals between them.* BP level fluctuates from a minute to minute, therefore it is recommended to find mean of 2-3 measurements with interval of 2 minutes. Record of BP value make with accuracy of 2 mm Hg. In the table 6 some factors influencing level SAP and DAP (Evidence-based Hypertension, BMJ Books, 2001) are given.

Table 6

Factors	Arterial pressure, mm Hg	
	Systolic	Diastolic
Absence of a support for the arm on which is measured BP	↑ 2	↑ 2
Absence of a support for the back	↑ 8	↑ 6-10
Crossed over feet	↑	↑
Small cuff	↓ 8	↑ 8
Cuff over clothes	↑ 50	↑ 50

Fast deflation of air from the cuff	↓	↑
Second air injection in the cuff	↑ 30-↓14	↑ 20- ↓ 10
Ambient noise	↓	↑
Cold indoors	↑ 11	↑ 8
Smoking	↑ 10	↑ 5
Talking of the patient	↑ 17	↑ 13
Phone talking	↑ 10	↑ 7
Overflow of intestine or bladder	↑ 27	↑ 22
Within 2 hours after coffee uses	↑ 10	↑ 8

At BP measuring except of SAP and DAP find following characteristics:

1. ***Mean pressure*** (fig.16) is the pressure expressing energy of the continuous motion of blood, which gives the same hemodynamic effect that is observed at natural fluctuating pressure of blood. One of the formulas of determination of mean pressure: $p_m = p_d + 1/3(p_s - p_d)$.

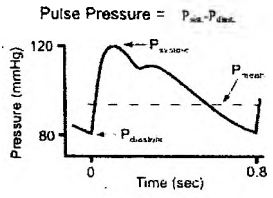


Fig. 16

2. Systolic (peak) BP expresses all reserve of a potential and kinetic energy of mass of blood on the given field. The systolic pressure develops from a so-called *lateral* systolic pressure and *shock* (hemodynamic impuct). The lateral systolic pressure is a pressure actually reacting in the systole on a lateral side of an artery. Hemodynamic impuct (H I) is framed at subitaneous occurrence of a hindrance before moving flow in the blood vessel, thus the kinetic energy transfers to the short moment in pressure (hemodynamic impuct). H I is result of activity of the inertia forces defined as a pressure rise when the vessel is squeezed. H I value at a healthy person is equal 10-20 mm Hg. *The true pulse pressure* is the difference between lateral and DAP. As value of H I is in advance unknown, find ***pulse pressure***, that is difference between the SAP and DAP. Pulse pressure typically ranges between 40-50 mmHg.

Order of work

Task 1

By Riva-Rocci method measure 3 times the SAP and DAP (if it is possible), give the interval estimation of the SAP value. At the same person measure 3 times SAP and DAP by Korotkov's method and then by oscillometric method according to the standard and give interval estimation. Calculate the mean and pulse pressure. Compare results, draw a conclusion, data place in the table.

Measuring method	Sistolic press., mm Hg, $p_s = \overline{p}_s \pm \Delta p_s$	Diastolic pres., mm Hg, $p_d = \overline{p}_d \pm \Delta p_d$	Pulse pressure, mm Hg, $\overline{p}_p = \overline{p}_s - \overline{p}_d$	Average pres., mm Hg, $p_m = p_d + 1/3(p_s - p_d)$
Riva-Rocci				
Korotkov				
Oscillometric				

Task 2
Impose the cuff over clothes, measure 3 times SAP and DAP by Korotkov's method, find a mean, compare to indications on the unclothed arm. Measure SAP and DAP some times on horizontal outstretched arm (isometric muscle tension). Draw a conclusion.

Task 3
According to the point 1 calculate possible value SAP at head level (h=0.5) and hip level (h=1).

Task 4 (by instructions of the teacher)

1) Measure SAP and DAP at 6 students on the left and right arms.
2) By means of the Shapiro-Wilk test check samples on normality.
3) Compare means SAP on arms on the Student test in case of normal samples, or by means of the test of Mann-Whitney if at least one of samples does not submit to the normal distribution law.

Control questions

1) What are the reasons of the continuous blood motion on the vascular system?
2) Formulate the stream continuity condition, write down and illustrate Bernoulli's equation and equation of Poiseuille. On what physical factors depends pressure in any point of vascular system?
3) What are the advantages and disadvantages of the direct method of BP measurement?
4) Biophysical bases of the methods of Korotkov and Riva-Rocci.
5) What are the key rules and conditions of correct BP measurement by Korotkov's method?
6) Give the short characteristics of indirect methods of BP measurement.
7) Characterise errors of the cuff methods of BP measurement.
8) What means grade accuracy B/B of a device of BP measurement?
9) What does characterize the average and pulse pressure?

LABORATORY WORK №14

Determination of the surface tension of liquids by the method of measuring of a maximal pressure in the air bubble

Purpose of the work: 1) to familiarize with one of methods of determination of the surface tension.

Equipment: 1) installation for determination of a surface tension,

2) vessels with distilled water and solutions of spirit of different concentrations.

Theory of the work

Origin of the surface tension force. Definition of the coefficient of the surface tension

On a molecule *a* (fig. 1) that is inside of liquid acts equal forces from the part of enclosing molecules. Resultant of forces is equal to zero. For molecules b, c, d situated near of the liquid surface, forces operating from the part of molecules of liquid exceed the forces operating from the part of molecules of air. As result there is resultant of forces F directional inside of liquid. Therefore for transition of a molecule from depth of the liquid to the surface layer is required to make some work, as molecules of the surface layer have the greater potential energy.

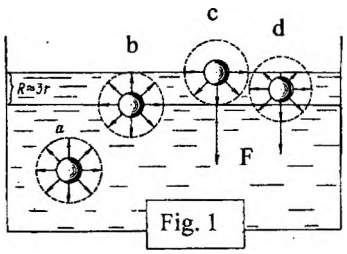


Fig. 1

The surface of a liquid behaves similarly to an elastic film that aspires to be reduced. The surface of a full-sphere has the least square and therefore the least energy. For example: airborne particles have the spherical shape.

The work ΔA required for formation unity of a liquid surface is named ***the surface tension*** α*:*

$$\alpha = \frac{\Delta A}{\Delta S}$$

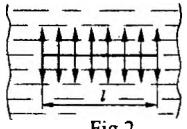


Fig.2

Let's select conventionally on the surface of a liquid the segment of length l (fig. 2). The surface tension force F is directed *perpendicularly* to the segment l and **on tangent to the liquid surface**. *The surface tension force F operating per unit of length of a contour of a liquid surface also is known* ***as the surface tension (or coefficient of the surface tension*** α*):*

$$\alpha = \frac{F}{l}$$

Unit of α: $J/\ m^2$ or $N/\ m$.

The surface tension α ***depends***: 1) on a sort of liquid, 2) on presence of impurities in a liquid, 3) on temperature T (with increase of T the surface tension α decreases).

The surface tension of biological liquids in some cases is ***the diagnostic factor***. So, at the icterus surface tension α of urine sharply decreases because of occurrence in urine of cholic acids; at diabetum α also sharply changes because of raise of content of lipase in the human blood.

Pressure under a curved surface of a liquid. Laplace's formula

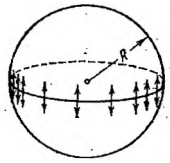
Tendency of a liquid surface to reduction result in pressure under a curved surface appears different than under flat surface. Under the convex surface pressure is more on value Δp and under the concave it is less on the same magnitude than under a flat surface (fig. 3).

Fig. 3

Let's calculate additional pressure Δp for a spherical surface. Let's dissect mentally a spherical drop of a liquid of radius R on two parts (fig. 4). The surface tension the surface layers of hemispheres are attracted each other with the force of $F = 2\pi R\alpha$. This force holds down hemispheres on a surface of square $S = \pi R^2$ and creates the additional pressure

$$\Delta p = \frac{F}{S} = \frac{2\pi R\alpha}{\pi r^2} = \frac{2\alpha}{R}.$$

Fig. 4



This formula is known ***as the Laplace's formula***.

Surface-active substances

Surface tension strongly depends on the impurities presented in liquids. Minimum of a surface energy is attained not only by decreasing of its surface, but also by completion of the surface layer by such molecules, that α was minimum. Materials adsorbing on the surface of a liquid and reducing its surface tension are called **surface-active (SAS)**. Molecules of such materials, first of all, act in the surface layer and only after they densely enough fills the surface layer, they penetrates into the basic volume of liquid. The most known of SAS are *the soap, spirit, ether* (in relation to water). To SAS usually belong molecules of the organic matters having the prolated shape. They consist of a polar group (for example, -OH,-COOH), to which joins not polar part of a molecule (for example, hydrocarbonic chain). In the surface layer SAS are drawn up parallel to each other, by polar groups inside of a liquid and by not polar outside.

SAS have wide application in medicine at preparation of the medical instrument, catgut, antibiotics and vitamins.

Our lungs are covered by layer of SAS, so-called surfactant (surfactant is the abbreviation for surface active agent), that hinders to adhesion of alveoluses at expiration and to the strong tension at inspiration. If at a child at birth is not present SAS in lungs, he cannot make the first breath (asphyxia) and can die. Development of surfactants in lungs depends on smoking, harmful impurities in air and also water in lungs, washing away surfactants.

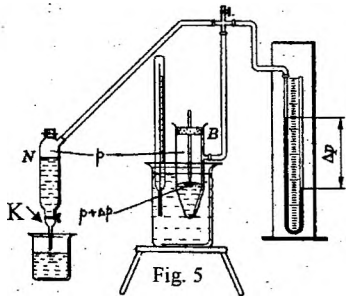


Fig. 5

Exposition of installation

For determination of α in the given work *the method of measuring of maximal pressure in air bubble is used.* Device for measurement of α designed by P.A. Rebinder. The device consist of a vessel (aspirator) N connected with the U-tube manometer and with closed vessel B with the explored liquid (fig. 5). The capillary touching a surface of liquid is inserted into the vessel B. If to open tap K, because of outflow of water pressure in the vessels N and B decreases. At some difference of pressures (between atmospheric p_0 and pressure above the surface of liquid in the tube p) in the liquid through the capillary will be forced a bubble: $p_0 = p + \Delta p$, where Δp is an overpressure in the bubble, stipulated by the surface tension and measured by the manometer.

First will carry out the measuring of Δp_0 for reference liquid (distilled water) with the surface tension of α_0:

$$\Delta p_0 = \frac{2\alpha_0}{R} = \rho g \Delta h_0,$$

where $\rho g \Delta h_0$ is hydrostatic pressure of a pole of manometer liquid of height h_0. Analogously for the explored fluid

$$\Delta p = \frac{2\alpha}{R} = \rho g \Delta h.$$

Having divided dextral and left-hand parts we shall receive the **working formula for α:**

$$\boxed{\alpha = \alpha_0 \frac{\Delta h}{\Delta h_0}}$$

Steps of the work

1). Pour some water into the aspirator and close the stopper; in the vessel B pour distilled water so, that the capillary only touched a surface of liquid.

2). Open the tap K so, that water dripped out with frequency ≈ 1 drop per second.

3). As soon as on the extremity of the capillary will appear a bubble note the indications of manometer Δh_0. Exchange distilled water by spirit and analogously find Δh for all solutions of spirit. Data put into the table.

4). Calculate α with help of working formula for different concentrations of spirit. Plot the graph $\alpha = f(c)$, using it discover unknown concentration of spirit.

Researched liquid (solutions of spirit)								Water	
								α_0,N/m	Δh_0,mm
c, %								$72 \cdot 10^{-3}$	
$\Delta h=h_1-h_2$, mm									
α, N/m									

Conclusion: 1) the surface tension depends on presence of impurities;

2) with increase of concentration of solution of spirit the surface tension α decreases, that is spirit is SAS in relation to water.

Control questions

1. Origin of the surface tension force. What is the coefficient of the surface tension α, on what it depends, units of α?
2. Proof of the Laplace's formula.
3. What are SAS?
4. Deduce of the working formula for α.
5. Where in medicine it is necessary to take into account the surface tension?

LABORATORY WORK №15

DETERMINATION OF VISCOSITY OF A LIQUID WITH HELP OF VISCOSIMETER

Purpose of the work: to master with one of the methods of determination of viscosity of liquid, to study dependence of viscosity of spirit on temperature.

Devices and accessories: unit for determination of viscosity by method of Poiseuille (Ostvald's viscosimeter), ethyl spirit, distilled water, thermometer, heater, researched liquid.

Theory of the work

All real liquids and gases have viscosity. Let two layers of a liquid or gas located from each other on distance dx have speeds ϑ_1 and ϑ_2 (fig. 1).

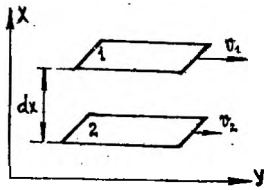


Fig. 1

On the side of the layer 1, which speed is more then speed of layer 2 ($\vartheta_1 > \vartheta_2$) force accelerating layer 2 operates. On the layer 1 retarding force operates on the side of layer 2. These forces are directed at the tangent to the surface of layer and are known as **forces of internal friction (viscosity)**. These are intermolecular forces.

Newton has established that force of internal friction F_{fr} between two layers of a liquid or gas is directly proportional to area S of adjoining layers and to gradient of speed $\frac{d\vartheta}{dx}$ (the value of $\frac{d\vartheta}{dx}$ shows how quickly speed changes at transition from layer to layer in the direction x perpendicular to speed of layers), **Newton's law**:

$$F_{fr} = \eta \frac{d\vartheta}{dx} S, \qquad (1)$$

where η is **viscosity** of a liquid (gas), η is **numerically equal** to force of internal friction arising between two layers at unit area and at gradient of speed equal to unit.

Unit of viscosity in SI is Pascal-second: Pa·s (1 Pa·s = 1N·s/m^2). Pa·s is big unit, therefore in practice viscosity of biological liquids can be measured in Poises (P), centipoises (cP): 1 Pa·s = 10 P, for example, for water η = 1 cP at temperature 20°C.

From the formula (1) we shall express viscosity: $\eta = \frac{F_{fr}/S}{d\vartheta/dx} = \frac{P}{d\vartheta/dx}$, i.e. viscosity is function of pressure and gradient of speed. Liquids for which viscosity does not depend on enclosed pressure P and gradient $d\vartheta/dx$ and depends only on temperature and the nature of liquid are called **newtonians**. For them η = const. To

newtonians liquids concerns *water, plasma of blood, low-molecular organic connections, true solutions, fused metals and their salts.* Coefficient of viscosity of other liquids (**high-molecular organic connections, suspensions, emulsions, blood**) depends on pressure and gradient of speed; it is **non-newtonian** liquids. The reasons of such behaviour are processes of aggregation, orientation and deformation of particles of non-newtonian liquid, i.e. because of presence of internal structure at such liquids. So, **blood being suspension of its uniform elements (erythrocytes, leukocytes, thrombocytes) in plasma is non-newtonian** liquid. Besides at movement of blood on vessels its uniform elements concentrate in the central part of flow for what viscosity in the central part of a vessel increases and in the layer near a wall of vessel impoverished by erythrotcytes viscosity is lowered (approximately in 1.5 times).

At pathologies viscosity of blood (in norm is 4 – 5 cP) changes at some diseases from 2 – 3 cP up to 10 – 15 cP for others. Besides *viscosity of plasma of blood are influences on ESR (erythrocyte sedimentation rate).* At inflammations ESR grows because of change of structure of plasma. Thus, viscosity can serve as trouble-shooting test at some diseases.

Various character of dependence of viscosity of liquids and gases on temperature (viscosity of gases grows with rise of temperature, η of liquids decreases) specifies the different mechanism of their internal friction.

Set of methods of measurement of viscosity is named **viscosimetry** and with the devices used at it are known as **viscosimeters.** Big range of values of viscosity (from 10^{-6} up 10^{12} Pa · s) and properties of researched mediums have caused the variety of methods of viscosimetry. We shall consider the method most frequently used in medicine.

Description of installation

Method of Poiseuille (capillary viscosimetry)

According to the **formula of Poiseuille** the volume of liquid V proceeding during time t through a tube of length ℓ at difference of pressure on the ends of a tube of ΔP is expressed by the formula:

$$V = \frac{\pi r^4 \Delta p t}{8 \eta \ell}, \qquad (2)$$

where r is radius of the tube, η is viscosity of liquid. Measurement of Δp, ℓ and r to lead inconveniently, therefore viscosity of researched liquid can be determined by comparison of its movement with the reference liquid (water), which viscosity is known, i.e. we use the relative method. For determination of viscosity with use of the formula (2) is necessary laminar flow of liquid. It is possible if the tube is sufficiently thin (capillary).

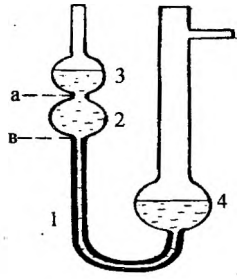


Fig.2

Ostvald's viscosimeter represents U-shaped glass tube (fig. 2), one of knee is *capillary* 1 from above finishing by *working* 2 and *auxiliary* 3 volumes. The certain volume of water (reference liquid) we pour into

the tank 4 and then having closed the right knee by the finger under pressure (with the help of a syringe) we lift water in the tank 3. Having removed the finger we observe downturn of the level. When the meniscus passes the label "a", we must switch on a stop watch; at passage of the label "b" we must stop and find t_0. Similarly find time t for the researched liquid (for different temperatures).

Then for equal volumes, for example water V_0 and spirit V is possible to write down: $V=V_0$ or $\frac{\pi r^4 \Delta p_0 t_0}{8\eta_0 \ell} = \frac{\pi r^4 \Delta p t}{8\eta \ell}$, whence reducing, we shall receive:

$$\frac{\Delta p_0 t_0}{\eta_0} = \frac{\Delta p t}{\eta}. \qquad (3)$$

Liquids in the capillary are moved under action of the hydrostatic pressure $\Delta p = \rho g \Delta h$ caused by the difference of levels in knees. Substituting in (3) $\Delta p_0 = \rho_0 g \Delta h$ and $\Delta p = \rho g \Delta h$ let's receive **working formula**:

$$\boxed{\eta = \eta_0 \frac{\rho t}{\rho_0 t_0}}. \qquad (4)$$

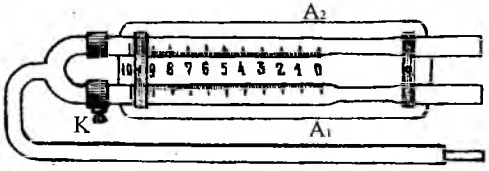


Fig. 3

Medical viscosimetr of Gess (BV-4) is used for measurement of viscosity of blood, it is based on application of formula of Poiseuille also. Viscosimeter BV-4 consists of two identical graduated pipettes A_1 and A_2, attached to a support. Inside the pipette pass capillaries of the identical sizes.

Into the pipette A_1 suck up the certain volume of water and close the tap K that allows collect in the pipette A_2 the researched liquid not changing the water level. If having closed tap K to create rarefied air, than displacement ℓ_0 of reference liquid and ℓ of researched will be inversely proportional to their viscosity:

$$\frac{\eta}{\eta_0} = \frac{\ell_0}{\ell} \quad \text{or} \quad \eta = \frac{\eta_0 \ell_0}{\ell}, \qquad (5)$$

where η is viscosity of the researched liquid, η_0 is viscosity of water.

It is possible to attribute to imperfection of capillary method stratifying of non-newtonian liquids because of unequal gradient of speed at walls and on the axis of capillary. Because of it capillary viscosimeter gives only *average value of viscosity*. Simplicity, accuracy, wide range are advantages (from 10^{-5} up 10^4 Pa·s) of this method.

Order of work

1. Determination of viscosity of spirit by Ostvald`s viscosimeter

1. Cautiously having washed out viscosimeter by distilled water fill by spirit expansion 4 up to half.

2. Vessel with cold water put on the heater and place viscosimeter in the vessel. Switch on the heater and each 5 ^{0}C measure time t of movement of spirit between labels "a" and "b" in the interval of temperatures 20 – 40 ^{0}C.
3. Wash out viscosimeter and do similar measurements for distilled water (i.e. find t_0).
4. Using tabulared values of viscosity of water η_0, density of spirit ρ and water ρ_0 (table 2) under the formula (4) find viscosity of spirit in interval 20 – 40 ^{0}C.
5. Results put in the table 1, construct the graph $\eta = f(t)$ for spirit, make the conclusions.

Table 1

t, ^{0}C	t, s	t_0, s	$\rho, \frac{kg}{m^3}$	$\rho_0, \frac{kg}{m^3}$	$\eta_0, 10^{-3}\ Pa \cdot s$	$\eta, 10^{-3}\ Pa \cdot s$

Conclusion: at increase of temperature viscosity of spirit decreases.

Dependence of density and viscosity of ethyl spirit and distilled water on temperature

Table 2

spirit		water	
t,^{0}C	ρ, g/cm^3	$\eta, 10^{-3}$ Pa·s	ρ_0, g/cm^3
15	0.7937	1.14	0.9991
20	0.7900	1.00	0.9982
25	0.7852	0.89	0.9970
30	0.7810	0.80	0.9956
35	0.7767	0.72	0.9940
40	0.7722	0.66	0.9922

2. Determination of viscosity of liquid by the medical viscosimeter BV-4 (to fulfill on demand of the teacher).

1. Open the tap K, the end of the right pipette lower in distilled water and having sucked up it up to zero mark, close the tap.
2. Researched liquid suck up in the second pipette up to the mark 0.
3. Open the tap K and cautiously suck up both liquids. Water and blood (or any researched liquid) will move in capillaries with different speeds. Position of water level in the capillary at achievement by researched

liquid of the label 1 shows viscosity of this liquid in relative units, i.e. η/η_0. Experiment repeat 3 times, find average value of $<\eta>$.

Control questions

1. Appearance of the forces of internal friction.
2. Write down the Newton equation for viscous liquid. What is gradient of speed?
3. Units of measurement of viscosity.
4. How does viscosity of liquids and gases depend on temperature?
5. What are newtonian and non-newtonian liquids?
6. Write down formula of Poiseuille.
7. Deduce the working formula for determination of viscosity by Ostvald`s method.
8. Measurement of viscosity by Gess viscosimeter.

LABORATORY WORK №16

DETERMINATION OF EMF BY THE METHOD OF COMPENSATION

Purpose of the work: to study compensation method of measurement of cell EMF; to measure the EMF of unknown galvanic cell and EMF between two points of the hand.

Equipment: galvanometer with zero division in the middle of scale; normal cell of Weston (E=1.018 V); explored cell; accumulator or rectifier; the slide wire of length 5 m; rheostat; unipolar and bipolar keys; electrodes; connecting wires.

Theory of the work

Let in a closed circuit the direct current flows (fig. 1a). In the homogeneous conductor connected with poles of a cell charges are moved under action of electric field from the points with larger potential φ_1 to the points with smaller potential φ_2. On fig. 1,b the potential distribution along a closed circuit (on the vertical the potential φ is postponed) is conventionally shown.

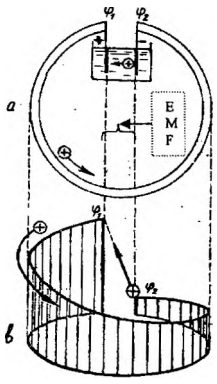


Fig.1

If on the charges acts electrostatic force (Coulomb force) only, then will be smoothing of potentials and current will be stopped (work in the electrostatic field on the closed path is equal to 0). Therefore for existence of electric current is necessary presence in a circuit of a device (cell), capable to support the potential difference due to action *of forces of not electrostatic origination (**extraneous (or side) forces**)*. The nature of extraneous forces can be different: in galvanic cells these forces originate due to energy of chemical reactions, in generator – due to mechanical energy of gyration of rotor, etc.

*The physical quantity equals to the work of extraneous forces at displacement of unit positive charge along the circuit is called **electromotive force (EMF)** E in the circuit:*

$$E = \frac{A_{ex.for}}{q} \qquad (1)$$

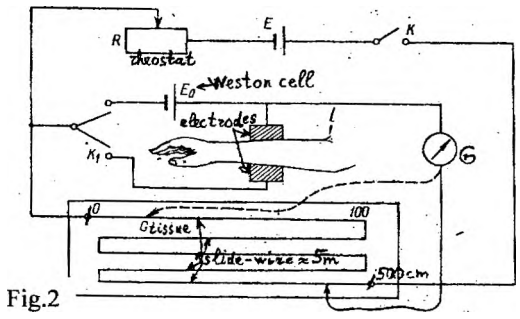


Fig.2

EMF is measured, as well as potential, in Volts (V).

Origin of EMF resulting in occurrence of a potential difference between two

points of tissues of a human organism was proved experimentally for a long time. Origin of EMF caused by contact with electrolyte (living tissue) of two heterogeneous electrodes or (at equal electrodes) by presence of different concentrations of electrolyte (on fig. 2 is shown the circuit of measuring of EMF originating between two points of a hand in our work). The potential difference between two points of a tissue can tend to tens millivolts (as for example, at ECG), and can change its magnitude within minutes or fractions of a second.

The part of a circuit is called *the nonuniform,* if except of Coulomb forces on this part extraneous force exists also, for example: the part 1-2 (fig. 3). Resistance of this part is R_{12}, and EMF E_{12} supports a potential difference $(\varphi_1 - \varphi_2)$. Direction of the current on this part can be anyone as the section 1-2 is a part of the circuit in which can be other cells. Let, for example, the current flows from the point 1 to the point 2. In time dt the charge dq=I dt flows past. Work of Coulomb forces dA_{Qoul} and extraneous forces $dA_{ex\ for}$ on transposition of a charge

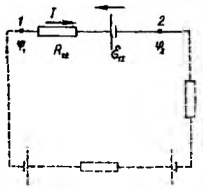


Fig. 3

$$dA_{Qoul} + dA_{ex\ for} = (\varphi_1 - \varphi_2)dq + E_{12}\,dq \qquad (2)$$

is equal to quantity of heat dQ produced in the part 1-2

$$dQ = I^2 R_{12}\,dt = I\,R_{12}\,dq\,. \qquad (3)$$

Equating right parts of (2) and (3), we shall receive:

$$I\,R_{12} = (\varphi_1 - \varphi_2) + E_{12} \quad \text{or} \qquad (4)$$

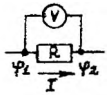
$$\boxed{I = \frac{(\varphi_1 - \varphi_2) + E_{12}}{R_{12}}}\,. \qquad (5)$$

The equality (5) expresses **Ohm's law for the nonuniform part of a circuit.**

The concept of voltage is generalization of concept "potential difference": voltage on the part 1-2 of the circuit is equal to the potential difference, if on this section does not act EMF, that is if E=0 then $U_{12} = (\varphi_1 - \varphi_2)$. In this case according to (5) we have **Ohm's law for the *homogeneous* part of a circuit** (fig. 4)**:**

$$I = \frac{\varphi_1 - \varphi_2}{R} = \frac{U}{R}\,. \qquad (6)$$

Fig. 4



If the electric circuit is closed, that is the ends of the part are connected then $\varphi_1 = \varphi_2$ and, as follows from (5) we have **Ohm's law for a closed circuit with EMF** (fig. 5):

$$I = \frac{E}{R_\Sigma}\,, \qquad (7)$$

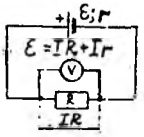


Fig. 5

$R_\Sigma = R + r$ is the total resistance of all circuit, R is resistance of the external circuit, r is internal resistance of a cell. Therefore the Ohm's law for a closed circuit we can write so

$$\boxed{I = \frac{E}{R + r}}\,. \qquad (8)$$

From (8) follows that E=IR + Ir, where $IR=U_{exter}$ is voltage on the external circuit, $Ir=U_{int}$ is voltage on the internal resistance of the cell (fig. 5). Therefore

$$U_{exter} = E - Ir = E - U_{int}, \qquad (9)$$

That is voltage on the external circuit is equal to EMF minus the voltage falling on the internal resistance of the cell. It speaks impossibility of measuring of EMF by the usual voltmeter: **it will measure voltage which is less of E on magnitude Ir.** The usual voltmeter demands for its work of presence of a current in the circuit. If the internal resistance of the voltmeter is great (for example, at a vacuum tube voltmeter), the current is $I \approx 0$ and the indication of the voltmeter approximately is equal to required EMF: $E \approx IR = U_{exter}$.

For more precise measurement of EMF is used **a method of compensation**. From (5) follows that *EMF a cell is equal to the potential difference on its poles only at absence of a current in it (I=0).* Absence of a current can be achieved, ***if EMF of a cell to compensate by a potential difference built by other cell. It is the basic idea of the method of compensation.***

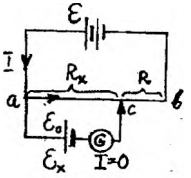


Fig. 6

The *basic circuit* of the method of compensation is shown on fig. 6. On this fig.: E is the constant-voltage cell ((accumulator or the rectifier); E_x is cell with unknown EMF; E_0 is normal Weston cell (E_0=1.018 V); G is galvanometer with null in the middle of scale; «ab» is the slide-wire with the movable contact "c", translocating which it is possible to achieve absence of a current in the galvanometer.

According to the basic idea of the method of compensation, voltage on the part of the slide wire R_x, equal to $U_x = I R_x$ (built by a current which is flowing past from the accumulator E) compensates EMF of the unknown cell $\mathbf{E_x = U_x}$. This equality is attained by transition of the slider of slide-wire ab while the galvanometer will show zero. Then on the foundation of (8)

$$I = \frac{E}{R_x + R}, \qquad (10)$$

Therefore

$$E_x = I R_x = \frac{E}{R_x + R} R_x. \qquad (11)$$

Then instead of the cell E_x we shall connect other cell with known EMF E_0 (normal cell of Weston) and again we shall achieve of compensation. Then, analogously

$$E_0 = I R_0 = \frac{E}{R_0 + R_1} R_0. \qquad (12)$$

Having divided (12) on (11) we shall receive

$$\frac{E_0}{E_x} = \frac{R_0}{R_x}, \quad \text{or} \quad E_x = E_0 \frac{R_x}{R_0} \qquad (13)$$

As resistance of the homogeneous section of slide-wire is proportional to its length $R = \rho \frac{l}{S}$, EMF of unknown cell is (**working formula**)

$$\boxed{E_x = E_0 \frac{l_x}{l_0}} \qquad (14)$$

That is for determination of EMF of unknown cell by the method of compensation is necessary to measure lengths of the slide-wire l_x and l_0 for which there is compensation for unknown and known cells.

Description of installation

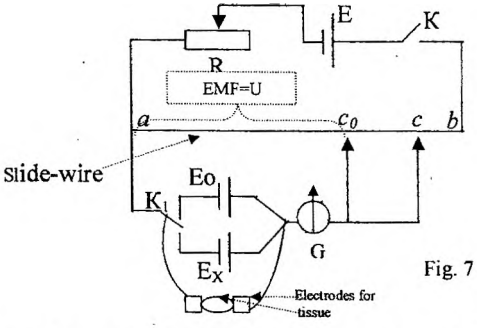
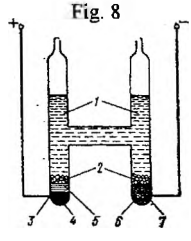


Fig. 7

The basic circuit differs from the experimental installation (fig. 7): in series with the cell E the rheostat R is included for regulation of current in accordance with decreasing of EMF of the cell E. Besides in series with the galvanometer high-resistance is switched on for decreasing of the current flowing through the sensing galvanometer; the key K is closed first, key K_1 serves for serial connection of E_x and E_0.

Fig. 8



Mercury-cadmium **normal Weston cell** (it is used as **etalon of EMF**) consists of a glass vessel, the shape and device which are shown on fig. 8. In the lower part soldered in electrodes from platinum, they are connected to terminals of the device. Mercury 4 serves as positive pole, the negative 6 is amalgam of cadmium HgCd. Electrolyte 1 is saturated solution of $CdSO_4$, and a depolarizer 5 is sulfuric oxide of mercury Hg_2SO_4 as pasta on which lay crystals of $CdSO_4$ - 4. At 20^0C E=1.0183 V. *The cell is necessary for preserving against percussions, and also it is impossible to use currents more than 10^{-5}-10^{-6} A.*

Order of work

1. Collect installation under the circuit (fig. 7).
2. Having closed the circuit by the key K and then having switched K_1 on the cell E_x, by moving of the slide to achieve of absence of current through the galvanometer G. Measure length of slide-wire ℓ, the voltage on which compensates E_x.
3. By switch K_1 to close the circuit on E_0 and moving the slide achieve of absence of current through the galvanometer. Measure length of slide-wire ℓ_0.
3. Repeat measurements of items 2 and 3 three times.
4. Data of measurements bring in the table:

EMF	n	ℓ	$<\ell>$	ℓ_0	$<\ell_0>$	E_X	E_0
Reference	1 2 3						
Researched	1 2 3						

5. Having averaged the data of direct measurements, under the formula (14) calculate value of unknown EMF E_x.
6. Analogously determine EMF originating between two electrodes, affixed to two places of a hand (fig. 2).
7. Make statistical processing of experimental results with confidential probability 0.95.

Control questions

1. What is work of electrostatic forces along the closed circuit? What is EMF? Units of its measurement.
2. Write down the Ohm's law for a nonuniform part of circuit, for a part (homogeneous) of circuit and for a closed circuit.
3. Why is impossible to measure exact value of EMF with the help of a voltmeter?
4. Give the basic idea of method of compensation. Deduce the working formula for determination of EMF of unknown source. What value does compensate EMF of cells E_x and E_0?
5. Draw the scheme of the experimental installation. Function of basic elements of the scheme.

Laboratory work № 17

STUDY OF THE ELECTRIC FIELD OF DIPOLE

Purpose of the work: construction of lines of equal potential of a current dipole and acquaintance with physical bases of Einthoven's theory.
Equipment: the flat bath with the coordinate grid, voltmeter with the probe, a dipole : two coal cylindrical electrodes+ transformer.

Theory of the work

The electric field is a kind of matter by means of which interaction between charges is carried out. *The force characteristic* of the electric field is *intensity* E (or ***electric field strength***): $\vec{E} = \vec{F}/q$, [N/C=V/m], where $\vec{F}$ is the force working on the unitary positive charge q placed in the given point of the field. Electric field can be presented by means *of lines of intensity* which has been lead so, that the tangent in any point coincides with the direction of the vector $\vec{E}$.

Potential is the power characteristic of the electric field: $\varphi = A/q$, [V], where A is work of the field on moving of the unitary positive charge from the given point to a point where the potential is accepted equal to zero. The lines which are taking place through the points with identical potential are called ***equipotential lines*** *(fig. 1)*.

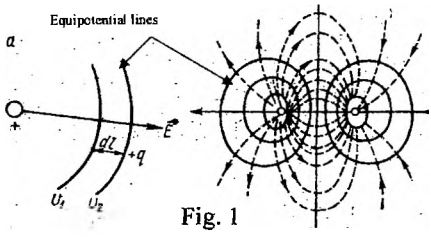


Fig. 1

Let's establish dependence between E and φ. Work of the field dA on moving of the charge q on the small distance dl along the line of intensity is equal dA=F dl = qE dl. On the other hand, this work is made by forces of the field due to reduction of potential energy of the charge dA = q dφ. Equating, we receive: qE dl = q dφ, whence E = - dφ/dl = - grad φ. The sign "minus" means that vector E is directed aside decrease of potential (dotted lines on the fig. 1).

Study of the electric field created by charges is carried out by finding of potentials in the different points of the field, because potential is easy for measurement. And, both in dielectric and in the conducting medium the electric field has similar character, but in the conducting medium is determined more easily. Therefore for study of the field of a dipole the bath is filled by conducting medium (water-supply).

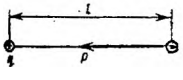


Fig. 2a

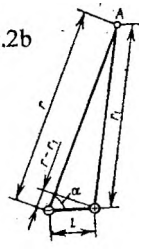


Fig.2b

Electric dipole is the system consisting from two dot equal on value but opposite on sign charges located on distance l from each other (fig. 2a).

Value $p = q \cdot l$ is the ***dipole moment.*** $\vec{p}$ is a vector directed from the negative charge to positive. We shall find potential in the point A (fig. 2b), located from charges of the dipole on distance r and r_1:

$$\varphi = \frac{q}{4\pi\varepsilon\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r}\right) = \frac{q}{4\pi\varepsilon\varepsilon_0}\left(\frac{r - r_1}{r_1 r}\right), \qquad (1)$$

where ε is relative dielectric permeability of the medium, ε_0 is electric constant. Let $l \ll r$ and r_1, then $r \cdot r_1 \approx r^2$, $r - r_1 \approx l\cos\alpha$, whence

$$\varphi = \frac{ql\cos\alpha}{4\pi\varepsilon\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{p\cos\alpha}{r^2} \qquad (2)$$

It is possible to show, that a potential difference between points A and B, taking place on equal distances from the center of a dipole is directly proportional to the projection of the vector p on the line connecting these points:

$(\varphi_A - \varphi_B) \sim p\ cos\alpha$. If the dipole is in the center *of the equilateral* triangle (fig.3) parities between voltages on the sides of this triangle can be received as the parity of projections of the vector p on the sides of the triangle:

$U_{AB}:U_{BC}:U_{AC} = p_{AB} : p_{BC} : p_{AC}$ or $U_{AB}:U_{BC}:U_{AC} = p\cos\alpha_{AB} : p\cos\alpha_{BC} : p\cos\alpha_{AC}$, (3)

where α is the angle between the dipole and the corresponding side of the triangle. As p is a constant last expression can be copied as Fig

$$U_{AB}:U_{BC}:U_{AC} = |\cos\alpha_{AB}| : |\cos\alpha_{BC}| : |\cos\alpha_{AC}|. \qquad (4)$$

In vacuum or isolator the electric dipole can be kept for a long time. In the conducting medium (for example, in a human body) there is a movement of free charges and the dipole either will be neutralized or shielded. At connection to our dipole any source of constant voltage the dipole will be kept, despite of presence of a current in the conducting medium. Such bipolar system is known *as the current dipole* or the ***dipole electric generator.***

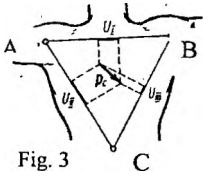


Fig. 3

Between a current dipole and an electric dipole there is the analogy based on the general analogy of the electric field in the conducting medium with the electrostatic field:

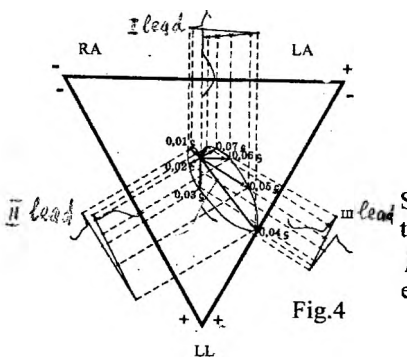


Fig.4

1) lines of current in the conducting medium coincides with lines of intensity of the electrostatic field at the identical form of electrodes;
2) many formulas describing fields have the identical kind at replacement q on I, $\varepsilon\varepsilon_0$ on $1/\rho = \gamma$ (specific conductivity).

Similarly to electric moment of a dipole is defined the ***dipole moment of a current dipole:***

$\vec{p} = I \cdot \vec{l}$, where l is the distance between electrodes, I is the current.

The *potential* of a field of a *current dipole* is expressed by the formula similar (2):

$$\varphi = \frac{1}{4\pi\gamma}\frac{p\cos\alpha}{r^2}. \qquad (5)$$

For a current dipole is fairly expression (4).

In the electric relation ***heart*** *is possible to consider as the* ***current dipole****. During an intimate cycle changes both direction of a vector p and its size.*

These positions are the **basic idea** of Einthoven's theory (Einthoven is the father of ECG).

According to **Einthoven's theory** heart is a current dipole located in the center of the equilateral triangle which tops are in the right arm, left arm and the left foot (fig. 3, fig. 4). Einthoven has suggested to measure differences of biopotentials of heart between tops of the equilateral triangle. On terminology of physiologists, the *difference of biopotentials between two points of a body* is known as **lead**. Distinguish the ***first lead*** (the right hand – the left hand RA-LA), the ***second lead*** (the right arm – the left arm RA-LL) and the ***third lead*** (the left arm – the left leg LA-LL). According to the formula (3), measurement of a potential difference between tops of this triangle allows to determine parity between projections of the dipole moment of heart on the sides of a triangle.

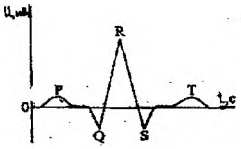


Fig. 5

Graph of dependence of a voltage on time U=f (t) in any lead is known as the ***electrocardiogram*** (fig. 5).

Description of installation

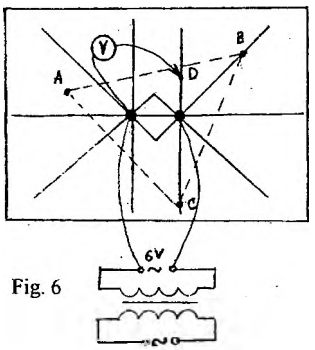


Fig. 6

Installation for carrying out of work represents a bath with water in which the measuring grid (fig. 6) is enclosed. The voltage from the lowering transformer gives on two coal electrodes, placed in the bath. Electrodes can move along the straight line. Measurements are made by the voltmeter with special probes. Not distilled water is a conducting medium, therefore a potential difference it is possible to measure between any points of our current dipole.

Order of work

I. Construction of equipotential lines of the current dipole field (alternating current).

1) Place electrodes on the centers of coordinates of the measuring grid according to figure 6 and switch on the transformer;

2) place the probe of the voltmeter near of one of electrodes in the point D. Moving the probe around of the electrode, find points with the same potential as well as in the point D. Connect the received points and you will receive an equipotential line;
3) construct around of each electrode two equipotential lines similarly;
4) lead force lines of the field of the current dipole perpendicularly to equipotential.

Conclusion: we have studied topography of the field of the current dipole: position of equipotential lines depends on size of the dipole moment, electrodes and properties of medium.

II. Check of a parity between projections of the dipole moment on the sides of the equilateral triangle (Einthoven's triangle).

1) Disconnect the probe of the voltmeter from the transformer;
2) not changing position of electrodes of the dipole, measure voltage on the sides of the triangle ABC: U_{AB}, U_{BC}, U_{AC};
3) determine $\cos\alpha_{AB}$, $\cos\alpha_{BC}$, $\cos\alpha_{AC}$. Check up the parity (4).

Conclusion: voltage in the leads are directly proportional to projections of the dipole moment on the sides of triangle of Einthoven: $U \sim \cos\alpha$.

III. Modeling of elements of the electrocardiogram (I lead).

1) Establish electrodes in the tops A and B of the triangle ABC (I lead);
2) Change the distance l between electrodes of the current dipole according to the table and write down U_{AB} for each value of l.
3) Construct the graph $U_I=f(t)$.

t	1	2	3	4	5	6	7	8	9	10	11
l, cm	4.6	5.6	4.6	4.6	3.4	8.6	4.6	3.4	4.6	5.6	4.6
U_{AB}											

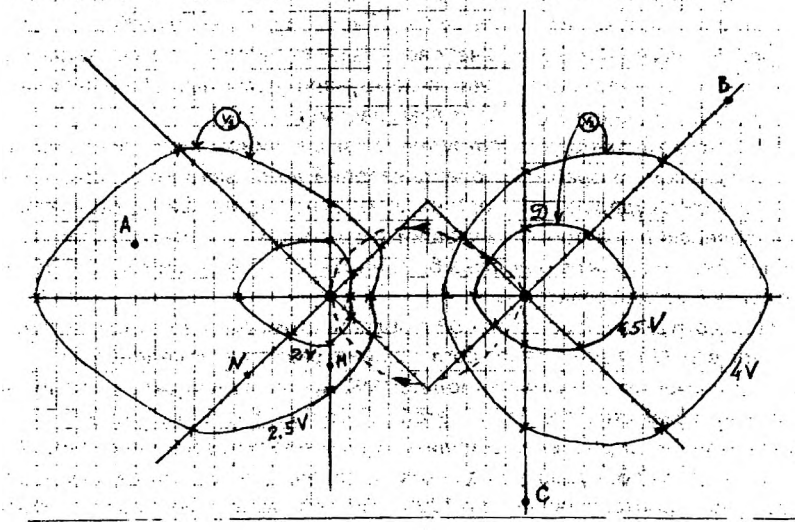
Conclusion: at change of distance between electrodes of the current dipole the picture of equipotential lines of the electric field changes and consequently the potential difference in corresponding lead changes.

Control questions

1) Define the intensity and potential of electric field; connection between them. Units of measurement of E and φ.
2) What lines are named equipotential? Why they are not crossed? How they are located in relation to lines of intensity? Deduce the formula for calculation of potential in any point A of the dipole. On what it depends?
3) What is the ratio of voltages on the sides of the equilateral triangle?
4) What is the current dipole? Why the charging dipole can be replaced by the current dipole at modeling of an electrocardiogram?
5) What are the basic postulates of Einthoven's theory?
6) What is the lead? Name three leads.
7) What is the electrocardiogram?

8) What will show voltmeter V_1 and V_2? (fig. 7). Compare electric field strength E_M and E_N in the points M and N (which of them is more? why?).

Fig. 7



LABORATORY WORK № 18

DETERMINATION OF RESISTANCE OF THE PART OF A HUMAN BODY TO THE DIRECT AND ALTERNATING CURRENT

Purpose of the work: to familiarize with features of passage of direct and alternating current through a live tissue.

Devices: microammeter of the direct and alternating current, source of a direct current on 6-12 V, voltmeter of the direct current, two not polarized electrodes, rheostat on 1-2 kOm, key, sound generator, connecting wires.

Theory of the work

It has been established that at passage of the direct current through human tissues the current does not remain constant (at the constant voltage). The current considerably decreases and after a while (fig. 1) is established at a constant level. This phenomenon is caused by presence of polarizing capacity, as result of electrochemical ***polarization***. At passage of an electric current through a live tissue in it arises *EMF of polarization* opposite directed to the enclosed voltage and this EMF reduces value of the current. Then according to the Ohm's law: I=(U-E(t))/R, where E (t) is polarization EMF, time-dependent.

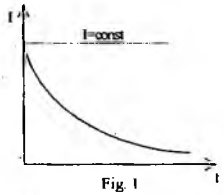


Fig. 1

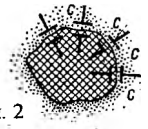
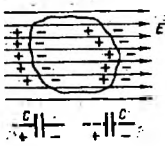


Fig. 2

Live tissues do not possess inductance, and *resistance of them has the active and capacitor components* caused by some structural components. Tissue of the human organism consists of the cells washed by a tissue liquid. Such element represents two mediums well conducting a current (a tissue liquid and cytoplasm), divided by badly conducting layer of cellular membrane (dielectric) (fig. 2).



All this gives to the tissues capacitor properties. Therefore on alternating current formula for total resistance Z at connection of R and C in series looks like:

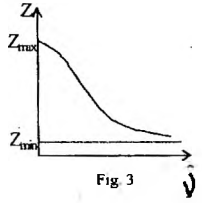


Fig. 3

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega c}\right)^2} \qquad (1)$$

From the formula (1) follows that at passage of alternating current through live tissues the *dispersion of electroconductivity* is observed. Total resistance of a tissue increases with reduction of frequency up to some value Z_{max} and also tends to some minimal value Z_{min} at increase of frequency (fig.

3). Dispersion of electroconductivity is inherent only for live tissues, at dying of tissues the steepness of the curve decreases and for the dead tissue this phenomenon is not observed. Frequency dependence of impedance of a tissue can be used as one of the tests, allowing to estimate *its viability at transplantation.*

The impedance of a live tissue depends on its physiological condition. So, at blood filling of vessels the impedance changes depending on a condition of cardiovascular activity. The *diagnostic method based on registration of changes of impedance of tissues, caused by change of blood filling is called rheography or the impedance - pletizmography.*

On electric properties tissues of an organism represent different mediums: *blood, liquid of spinal cord, intercellular liquid has small resistance, they are conductors* of a current. The big resistance have the *bone, dry skin, fatty tissues, they are dielectrics*. Electroconductivity of a part of a body strongly depends on resistance of a layer of *skin* under electrodes. Resistance of skin depends on its condition: humidity and thickness of the horn layer. Therefore resistance of a skin can change from tens up to hundreds thousand Ohm. Having overcome resistance of a skin, current goes *on the way of the* ***least*** *resistance*: blood vessels, muscles, nerves. Resistance of internal tissues is small: 300 – 500 Ohm.

Taking into account that total resistance (impedance) for alternating current is defined only by active resistance and capacity, combining them, it is possible to make electric models of biological objects as *equivalent electric circuits*. The most simple are circuits with connection of R and C in series (fig. 4a) and in parallel (fig. 4b) and more complex is the circuit on fig. 4c. For the circuits represented on fig 4, frequency dependence of impedance is submitted on fig. 5 (a, b, c).

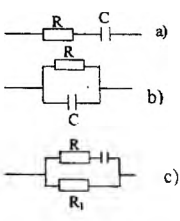


Fig. 4

From the graphs of equivalent circuits (fig. 4a, b) is possible to draw the conclusion, that they cannot be to the full applied for modeling of live tissues as they do not satisfy to all range of frequencies and do not correspond to the skilled data. So, for the circuit with parallel connections of R and C the impedance on the big frequencies tends to zero, at real objects the impedance is reduced only up to the certain value Z_{min}. Only the circuit on fig. 5c correctly reflects resistance of a tissue on big and small frequencies.

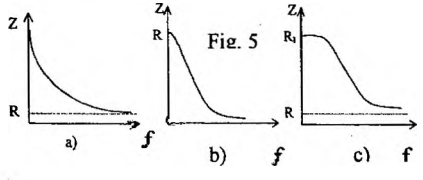


Fig. 5

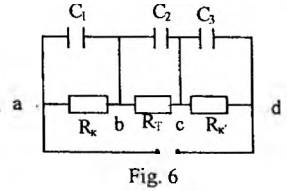


Fig. 6

Distinctions at passage of the direct and alternating current through a live tissue can be explained on equivalent electric circuit of a live tissue (fig. 6), on which $R_к$ и $R_к'$ are resistances of the skin; $R_т$ is resistance of the tissue under the skin. Resistances $R_к$, $R_т$, $R_к'$ are shunted accordingly by

capacities C_1, C_2, C_3. Direct current will pass only through the specified resistance: $R = R_k + R_T + R_k'$. Alternating current on the subcircuit **ab** and **cd** will pass through shunting capacities C_1 and C_3 (R_k и R_k' will have for it the big resistance) and basically resistance to alternating current will correspond to the subcircuit **bc**, consisting of R_T and $R_C = 1/\omega c$.

Description of installation

For determination of resistance of the part of a body on direct current the circuit (fig. 7) is used: K is the key; E is the source of direct current, R is rheostat included as potentiometer, T is a part of human body, V is voltmeter, μA is microammeter. Between the tissue and electrodes is placed gauze packing moistened by physiological solution.

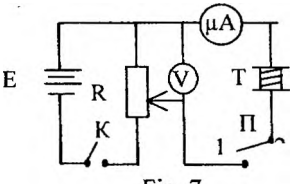


Fig. 7

At determination of resistance to alternating current is used the sound generator, on which output the microammeter and a body part with the electrodes imposed on him (fig. 8) are included.

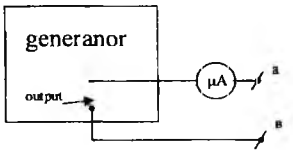


Fig. 8

Order of work

1. Determination of resistance of a part of human body and resistance of skin to the direct current.

1. Collect the circuit in conformity about fig. 7.
2. Establish the voltage 3V with the help of potentiometer. Impose electrodes on the hand. Write down the current corresponding to the given voltage. Similar measurements do for 4 and 5 volt.
3. At small distance between electrodes $R_k = R_k'$ and $R_k >> R_T$. Therefore resistance of the part between electrodes is equal $R_X = R_k + R_T + R_k' = 2R_k$. As $R_X = U/I$, then resistance of skin will be equal to $R_k = R_x/2 = (U/I)/2$.
4. Results of measurements bring in the table 1:

№	I, A	U, V	R_X, Ohm	$\langle R_X \rangle$, Ohm	$\langle R_k \rangle$

CONCLUSION: *resistance of skin on the direct current has significant value and depends on a condition of skin of a person.*

2. Study of frequency dependence of impedance of a live tissue.

1. Collect the circuit in conformity about fig. 8. Switch on the generator and establish the voltage 4 V and frequency 50 Hz.
2. Apply two electrodes on the hand and write down the current. Similarly determine a current on frequencies of 500, 1000, 2000, 5000, 10000, 20000 Hz.

3. Under the formula Z=U/I determine impedance of tissues on each frequency. Construct the graph Z=f (ν). Data bring in the table 2.

4. Under the graph by extrapolation for $\nu \to \infty$ determine value of active resistance of the tissue R_T. Ratio $\cos \varphi = R_T/Z$ characterizes the phase-shift angle between a current and the voltage enclosed to a tissue (determine $\cos \varphi$ for frequency of 50 Hz).

Table 2

№	ν, Hz	U, V	I, A	Z_{min}, Ohm	$\cos \varphi$

***Conclusion**: with growth of frequency impedance of a tissue decreases (according to (1)) and tends to R_T. Resistance of a tissue on alternating current is less than on direct at the same voltage.*

Control questions

1. What are the features of electroconductivity of a live tissue on direct current?
2. What is the nature of capacitor properties of tissues?
3. Why for alternating current between I and U for live tissue there is phase-shift angle?
4. Write down the formula of impedance of a live tissue for the equivalent circuit with connection of R and C in series.
5. What is the dispersion of electroconductivity? What for it needs to know in medicine?
6. What is equivalent circuit? Draw the possible variants of equivalent circuits and graphs Z=f(ν) corresponding to them.
7. How does determine the phase-shift angle with the help of graph of dispersion of electroconductivity?

Laboratory work № 19

DETERMINATION OF PARAMETERS OF ELECTRIC IMPULSES

Purpose of the work: to learn to determine parameters of pulse currents.
Devices and accessories: the device of stimulation of muscles (ACM), oscillograph.

Theory of the work

Now in medicine for influence on various systems of human organism are used electric pulse signals. This explains from the fact, that leading of energy of the physical factor to human organism in the separate portions divided by pauses allows not only to reduce the heat of tissues and electric loading on cardiovascular and nervous systems, but also to carry out *selective* influence on certain bodies and systems by selection of corresponding parametres of influence.

Electrical impulse is transient modification of an electrical voltage or a current against some constant value (fig. 1). Impulses are divided on two bunches: 1) video pulses: electrical impulses of a direct current or a voltage (they also are perceived in physiology the term «electrical impulse» - fig. 1), and 2) radio pulses: the modulated electromagnetic oscillations.

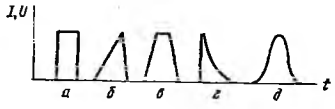


Fig. 1

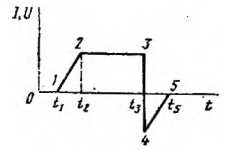


Fig. 2

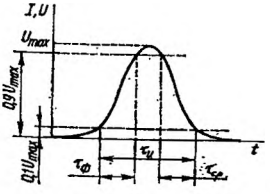


Fig. 3

Impulses under the shape happen: rectangular, sawtooth, trapezoidal, exponential, bell-shaped, etc.

Characteristic parts of impulse (fig. 2) are: 1-2 wavefront, 2-3 top, 3-4–back front, 4-5 wavetail. At it are accurately spotted the beginning moment t_1, transition from front to vertex t_2 and the impulse extremity t_5.

In a *real* impulse these times are not defined exactly (fig. 3), therefore their observational determination can give an essential error. For error decrease have agreed to choose time moments at which a voltage (current) have a value of 0.1 U_{max} and 0.9 U_{max}, where U_{max} is amplitude of U, $\tau_ф$ is duration of a front, τ_u is pulse duration, τ_{cp} is duration of a back front.

Impulse signal *key parameters* are: *amplitude of voltage* U_{max}, *pulse duration* τ_u and a *rate of pulse rise of front* $S_ф$:

$$S_д = \frac{0.9U\ \max - 0.1U\max}{\tau_д} = \frac{0.8U\max}{\tau_д}\ (1).$$

The impulse current are called repeated pulses (fig. 4).

***Key parametres* of a pulsing (impulse) current** (or voltage) (fig. 4) are: **period of recurrence T, frequency of recurrence** $\nu = \frac{1}{T}$ (2), **relative pulse duration** (dimensionless quantity showing in how many times period T exceeds duration of impulse $t_и$): $Q = \frac{T}{t_и}$ (3) and **duty factor**: $K = \frac{1}{Q} = \nu\tau_и$ (4).

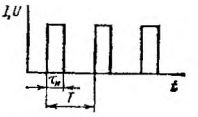


Fig. 4

The more rate of pulse rise of front of pulsing current, then at the smaller current comes traction (i. e. contraction of muscles): **Dubois-Reymonds'** law. It testifies that muscles adapt to current increase, there come compensatory processes at the expense of redistribution of corresponding ions in organism tissues.

Threshold current strength is a minimum current which forces muscles to be reduced (depends from tissue, frequency, place of contact of electrodes and others factors and approximately equals 1 mA for frequency of 50 Hz).

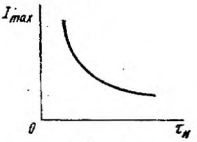


Fig. 5

There is a certain relationship between a threshold current I_{max} and duration of a rectangular impulse, that causes irritation (fig. 5). This relationship expresses

Weiss-Lapik's equation: $I_п = \frac{a}{t} + в$,

where "*a*" and "*в*" are constants (fig. 5). A constant "b", defining a *minimal threshold current necessary for irritation at long influence of current* ($t \to \infty$) is called **rheobase** (Greek rheos - flow). If both members of equation to increase on t, we will receive $I_п t = a + вt$ or $a = I_п t - вt$, i.e. "*a*" there is a charge, which is necessary for passing to cause excitation at very short time of influence.

Time t, necessary for irritation at the current equal two rheobases is named **chronaxia** (Greek - chronos - time, axia - measure). The value of chronaxia is an indicator of speed of occurrence of excitation and speaks about level of excitability of a tissue. The impulses corresponding to points located below a curve do not cause irritation. This curve is specific to different muscles.

If to increase frequency of a pulse current, keeping the excitation condition (area above the curve on fig. 5), at some frequency there will come long contraction (tetanus), it is similar to an alternating current of corresponding frequency.

Electrostimulation is *application of a pulse current for the purpose of excitation or strengthening of activity of certain bodies, muscles and nerves.*

Electrostimulation by video impulses is applied at the residual phenomena after a poliomyelitis, at neuritis of facial nerve, traumatic neuritis and paresis, which have developed in connection with long inactivity of muscles. For example, rectangular pulses (fig. 1a) strengthen brake processes in the central nervous

system and them are used for reception of the condition similar to sleep (electric sleep), and also at some mental diseases, hypertensive illness, a stomach ulcer.

Sawtooth impulses (fig. 1б) are applied to electrogymnastics of cross-striated muscles at disturbance of their functions.

Exponential impulses (fig. 1в) can cause a motor reaction of strongly affected muscles when sawtooth cannot make it.

Parametres of the pulse currents used at electrical stimulation:

1. **Amplitude of pulse current I_0** depends on an organ and it should exceed a threshold current, differently there will be no stimulation. On the other hand, the current should be less shock current (not releasing current). The amplitude of a current depends also on the size of electrodes, type of a tissue and other factors and it is in limits of I_0=1-50 mA.
2. **Duration of impulses** t_u should not be too small as it can lead to magnification of threshold current and should be some milliseconds.
3. Pulse **period T** should be more than a refractory period T_{refr} for the given tissue. **The refractory period** is a time during which muscles cannot be exitated a usual threshold current that defines frequency of a stimulating pulse current: $\nu_{stim} = \frac{1}{T} \le \frac{1}{T_{refr}}$. So, for *skeletal muscles* $T_{refr} \approx$ 5 ms; $\nu_{stim} = \frac{1}{T} \le \frac{1}{5ms} \le 200\ Hz$, for *a cardiac muscle* $T_{refr} \approx$ 300 ms and $\nu_{stim} < 3\ Hz$.

   Really in the apparatuses for electrical stimulation are used pulse currents of frequency 1-200 Hz. At higher frequencies stimulation can be too, but any more each of brought impulses will cause electrical stimulation: impulses which get to a refractory period do not cause the answer. On frequencies over 10 kHz the electrical stimulation is not effective.

Steps of the work

1). Set oscilloscope handholds on the labels , switch on the oscilloscope in to the mains.

2). On the apparatus of stimulation of muscles (ACM) set handholds: "Modulating frequency": in the position «Without modulation», «Current of a patient»: on the label.

Tumblers "Current": in the position 10 mA, «Rhythmic modulation»: without modulation, "Polarity": direct, «Frequency of impulses»: 60 Hz.

3). Connect ACM to the input **Y** of the oscilloscope, switch on ACM in to the mains.

4). On the oscilloscope screen should be a signal  ; by the handhold «Frequency continuously» on the oscilloscope stabilise the image.

5). Draw a sketch of the image of the gained signal in to the copy-book and using a screen grid, determine impulse signal parametres, including its voltage proportional to the corresponding sizes of the image (a monitor signal voltage is equal 2.5 V in the position "Attenuation" 1:1 and 0.25 V in the position 1:10). As

frequency of signals is known, then period T will be proportional to the segment ℓ_0 of distance between the next impulses on the screen across, hence, necessary times τ_ϕ, τ_u and τ_{cp} are possible to determine, using a ratio, for example: T corresponds to ℓ_0, τ_ϕ corresponds to ℓ, then $\tau_\phi = \frac{T \cdot \ell}{\ell_0}$ (5).

6). Set the tumbler  or  in the next position and do for the given signal item 5.

7). Calculate S, T, Q, K under formulas 1-5. Results bring in the table.

Parameters / Shape of impulse	ν, Hz	τ_ϕ, s	τ_u, s	τ_{cp}, s	U_{max}, v	T, s	Q	K	S, V/s
Rectangular									
Exponential									

Control questions

1. What is electrical impulse, impulse current?
2. What shapes of impulses do you know?
3. Name the characteristic parts of impulse.
4. Name the key parametres of impulse and a pulsing current.
5. How does calculate parameters of impulse and pulsing current?
6. What is physiological activity of a pulsing current? Formulate laws of Du bois-Reymond and Weiss-Lapik.
7. Applications of impulse currents in medicine.

Laboratory work №20

Study of the galvanizing apparatus

Purpose of the work: to study purpose and principle of action of separate elements of the rectifier, to familiarize with work of the device for galvanizing, with concept of the medicinal electrophoresis.

Devices and accessories: the unit for study of the full-wave rectifier, oscillograph, the galvanizing apparatus, two glass vessels with solutions of NaCl (0,85 %) and KI (5 %), electrodes, conductors with tips, cotton wool.

Theory of the work

Galvanizing is the method of medical influence on the human organism by direct electric current up to 50 mA and by the voltage till 80 V. Now for galvanizing is used exclusively direct current received by the way of rectification of alternating current. The amplitude should not exceed of 0.5 %.

In connection with the big specific resistance of a dry skin ($\approx 10^7$ Ohm·m and humidified is $2 \cdot 10^3$ Ohm·m) the current under electrodes passes in a tissues mainly through apertures of a sweat glands. Then, considerably branching and deviating from a straight line connecting two electrodes, the current passes *through tissues with smaller resistance*: on blood vessels, nervous and muscular fibres.

On a way of current on both sides of semi-permeability membranes, cellular environments there are a congestion of ions. Between such congestions of ions arises EMF of interstitial polarization. It creates additional resistance to the current and such sites are places of the most active action of current.

Meeting the big resistance of epidermis, energy of direct current in part turns to heat, which can cause weak biological effects as activization of blood circulation and amplification of biochemical processes. *The basic component of action of direct current is its influence on parity in tissues of the different ions and this parity defines functional condition of tissues.*

The certain value in the mechanism of medical action of direct current has the phenomenon of electroosmosis, when under action of current through a membrane passage of water amplifies. Therefore, under the cathode there is a hypostasis and loosening of tissues and under the anode there is their condensation.

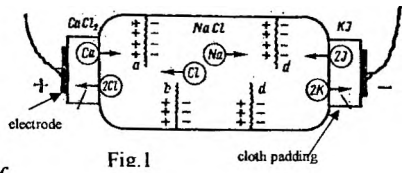


Fig.1

The direct current is used in medical practice also for *introduction of medicinal substances into a tissue of human organism through a skin or mucous membrane. This method is called* ***medicinal electrophoresis.***

Ions of medicinal substances are entered from the electrode of the same ***polarity.*** Ions of metals (K^+, Mg^{2+}, Mn^{2+}), vitamin E, B_1, B_{12} are entered from positive and ions of acid radicals, some organic connections are entered from the negative electrode.

To leading a current to the surface of body are applied flat electrodes from sheet lead by thickness of 0.3 – 0.5 mm or electrodes of the special form. As tissues contain a plenty of opposite charged ions, for example:NaCl ⇆ $Na^+ + Cl^-$, than at contact of the electrode with the body occurs electrolysis, ions turn to neutral atoms of Na and Cl, that incorporating with water forms the acid (HCl) at the anode and at the cathode alkali (KOH, NaOH). For exception of irritation or burns of a body by the acid or alkalis between a skin and an electrode is placed moistened in water and wring out *cloth padding.*

The basic circuit of galvanizing apparatus is shown on fig. 2.

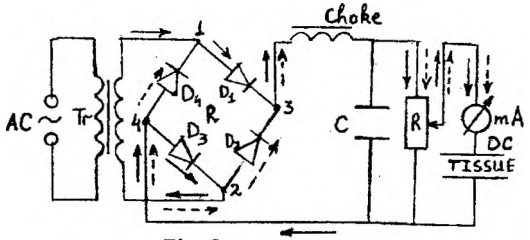


Fig. 2

Description of installation

For study of work of the rectifier is used the unit (circuit is resulted on fig. 3). Transformer TR lowers mains voltage up to necessary value. Diod bridge D consistы of 4 semi-conductor diodes having property of unilateral conductivity.

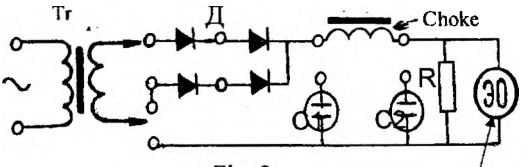


Fig. 3 Cathode ray tube

Two condensers C_1 and C_2 and the choke Др forms the filter that smooths pulsations of AC current. The choke represents the coil of inductance with the iron core. At passage through it of alternating current arises EMF of induction that interferes with change of a current. Condensers, being charged at the moment of increase of current and being unloaded at its reduction, also promote smoothing of pulsations. As result of simultaneous action of the choke and condensers the alternating current becomes the constant.

From the resistor R voltage puts on the input "Y" of the electronic oscillograph and on its screen is possible to see the process of rectification of the alternating current.

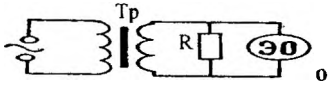


Fig. 4

Order of work

1. Reception and supervision of the oscillogram of alternating current.

Collect the circuit under the fig. 4 and draw oscillogram in your copy book (fig. 7, a).

2. Reception and supervision of the oscillogram of the half-wave rectification.

6. Collect the circuit according to fig. 5.

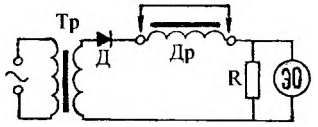


Fig. 5

7. For deenergizing of the choke connect the specified points by the wire. Draw the oscillogram in your copy book (fig. 7, b).

3. Reception and supervision of the oscillogram of the full-wave rectification.

1. Collect the circuit (see fig. 6).

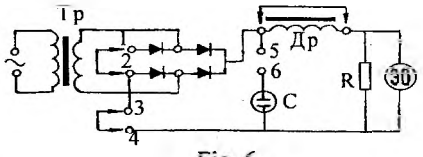


Fig. 6

2. Connect by the wire the points 1 and 2; 3 and 4.

3. Observe the oscillogram and copy it (fig. 7, c).

4. Supervision of action of the smoothing filter.

1. On the circuit (fig. 6) connect points 5 and 6 and observe the action of the condenser (fig. 7, d).

2. Remove the wire switched off the choke and observe its action on a pulsing current (fig. 7, e).

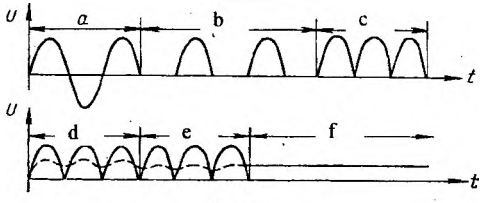


Fig. 7

3. Observe simultaneous action of the condenser and the choke (fig. 7, f), to make oscillograms in the copy book .

5. Acquaintance with the work of the galvanizing apparatus.

1. Preparation of the device for work:
   a) switch on the device in the network; put the toggle-switch of network in the position "on"-вкл, on the panel of the device the alarm bulb will light up. In 3 minutes the device will be ready to work;
   b) put the toggle-switch of limiting value of a measured current in the necessary position (50 mA) and determine the scale division value of milliammeter.
2. Check of polarity of the device:

a) thin tourniquet from cotton wool moisten by solution of KI, put it on glass and place above electrodes;
b) within several seconds to pass the current 40-50 mA, for this purpose the regulator of the potentiometer smoothly turn clockwise until the necessary current will be established;
c) by the regulator to reduce direct current up to 0 and to observe yellow cotton wool under the anode;
d) to draw a conclusion on correctness of the specified polarity.

3. Determination of a value of the minimal appreciable current through a tissue:
   a) put the toggle-switch of a limiting current to position 5 mA, determine the scale division value of milliammeter;
   b) establish the regulator of the potentiometer in zero position;
   c) put a finger on the electrodes (padding are moistened with solution of NaCl) and smoothly turn the regulator of the potentiometer, increasing value of current up to minimal appreciable value;
4. Introduction of iodine inside of NaCl solution.

a) In a glass with 0.85 % solution of NaCl lower the electrodes connected to the device of galvanizing. Thus the negative electrode should be wrapped up by the cotton wool moistened by KJ, the positive electrode wrap up clean cotton wool.

b) Passing a current 40-50 mA from the device of galvanizing 5-10 minutes observe typical for iodine yellow colour cotton wool at the anode, because ions of iodine pushing off from the same charged cathode have achieved the anode.

Control questions

1. What is galvanizing?
2. Why at galvanizing between the tissue and the electrodes we placed the cloth padding moistened by water?
3. What is the minimal appreciable current? Give it value.
4. What is the way and basic component of direct current under the electrodes?
5. What is the medicinal electrophoresis?
6. From what electrode are introduced ions at electrophoresis?
7. Draw the basic circuit of galvanization apparatus and specify assigning of basic elements.
8. How does determine the polarity of electrodes of galvanizing apparatus?

LABORATORY WORK №21

Study of dependence of resistance of a semiconductor on temperature

Purpose of the work: to investigate dependence of resistance of thermistor on temperature, to determine thermal coefficient of resistance, to find the temperature of the arm with the help of thermistor.

Devices and accessories: galvanometer, box resistance, rheohord, thermistor of some mark (MT-54, KMT-4, MMT-1) in a glass tube (in case of absence of it is possible to replace with the dot diode), thermometer with the scale division value of 1^0C, the source of direct current of 4-6 V, push-button key, connecting wires.

Theory of the work

At formation of a solid atoms or molecules of substance are approached to each other. Force of interaction between them grows. Therefore corresponding power levels of electrons of separate atoms and molecules approaches also, forming a **zone**. Thus the greatest change is undergone with levels of valent electrons from which the **valent zone** (*or valence band*) (fig. 1) is formed and by which electroconductivity of substance is caused. Alongside with power levels of valent electrons in a crystal there are so-called levels of excitation that at crystallization of substance forms a ***zone of conductivity*** with the higher values of energy than in the valent zone. Electron cannot proceed from the level of valent zone to any level of zone of conductivity without expense of energy from outside.

Zone of conductivity (or *conduction band*) at metals (fig. 1a) directly adjoins to the valent zone and even blocks it. At temperature of absolute zero valent electrons fill in levels of the valent zone before overlapping with zone of conductivity that is not filled by electrons. The distance between levels of zones, expressed in terms of energy is small value approximately equal to 10^{-23} eV (electron×Volt, 1eV=1.6 × 10^{-19} J). This energy is much less than value of thermal energy at room temperature ($\Delta E = \frac{3}{2}\kappa T = 0{,}04$ eV). To electrons of metals is enough to give from outside small energy for their transition to zone of conductivity. Therefore metals even at temperatures close to zero have good electroconductivity.

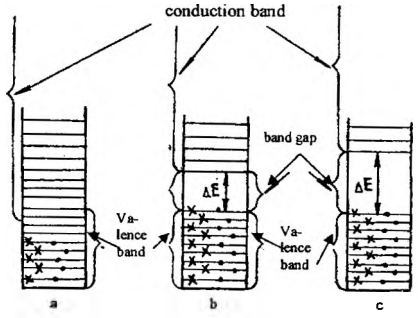


Fig. 1

Entirely filled by electrons the *valent zone of semiconductors* is separated from the zone of conductivity by the ***band gap*** (fig. 1b) which width in terms of

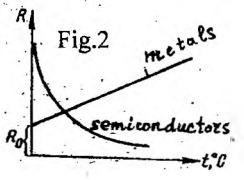
energy (depending on a kind of the semiconductor) can reach of 2 eV. Therefore for transition of electrons to the zone of conductivity and hence occurrence of electroconductivity of the semiconductor is necessary to give them from outside the additional energy approximately equal to 2 eV. Electroconductivity of pure semiconductors is named own conductivity.

Width of the band gap of dielectrics considerably exceeds value of 2 eV (fig. 1c), therefore for transition of electrons from the valent zone to the zone of conductivity is necessary to give them outside energy which value is not less value of energy of the band gap. Thus, semiconductors on electroconductivity borrow intermediate position between metals and dielectrics.

With rise of temperature resistance of metals changes under the linear law:

$$R=R_0(1+\alpha_м t), \qquad (1)$$

where R_0 is resistance at 0^0 C, α_m is the **thermal coefficient of resistance** describing *relative change of resistance at heating on one degree i.e.* $\alpha = \frac{R-R_0}{R_0 t} = \frac{1}{R_0}\frac{\Delta R}{\Delta t}$. It can be defined, having taken the derivative from (1) with respect to t, i.e.:



$$\alpha_м = \frac{1}{R_0}\frac{dR}{dt}. \qquad (2)$$

For pure metals in the interval of temperatures 0-100^0 C α_m is closed to 1/273 degree^{-1}.

Change of resistance of semi-conductor materials for temperatures that are not exceeding of 500 K is expressed by the formula:

$$R = Ae^{\Delta E/(2kT)}, \qquad (3)$$

where A is a constant dependent on geometry of the body and concentration of free carriers of charge, ΔE is the width of the forbidden zone, k is constant of Boltzmann, T is absolute temperature.

Having taken the derivative from R on T, from the formula (2) we shall find thermal coefficient of resistance of a semiconductor:

$$\alpha = -\Delta E/(2kT^2). \qquad (4)$$

Apparently from the formula (4), α_s is negative value. On absolute value it is more than α_{met} in 10 and more times.

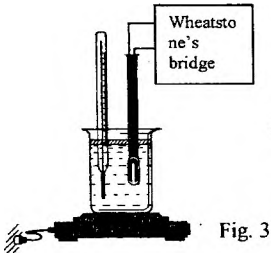
The width of the forbidden zone can be determined having taken two any resistance of semiconductor under the formula (3) for different temperatures (T_1 and T_2), then to divide them term by term and to find the logarithm:

$$\Delta E = \frac{2kT_1T_2}{T_2-T_1}\ln\frac{R_{T_2}}{R_{T_1}} = \frac{2\cdot 2{,}3kT_1T_2}{T_2-T_1}\lg\frac{R_{T_2}}{R_{T_1}}. \qquad (5)$$

Ability of a semiconductor considerably to change resistance with change of temperature is used at manufacturing of thermometers that applies in the different areas of science and technics, including biological and medical practice. Such devices are known as ***thermistors***. *The small sizes of thermistor allows to place it*

inside of a needle for measurement of temperature of hypodermic cellular tissue, muscular fibres or even cells, that is important at study of oxidizing processes.

Description of installation



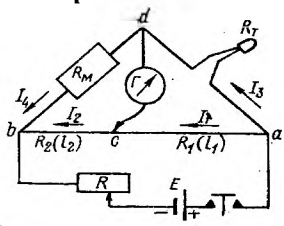


Fig. 3

For research of dependence of resistance of thermistor on temperature is used the bridge circuit (fig. 2) – ***Wheatstone's bridge***, in one of which shoulders is included thermistor R_T and in the second *resistance box* R_M. The bridge works from the source E of direct current. The voltage on rheohord *ba* is adjusted by rheostat R. Sites *the ac*, *ad*, *bc*, *bd* are called the *shoulders* of the bridge, *dc* is the diagonal of the bridge.

Galvanometer is switched in the diagonal. **Bridge is considered *counterbalanced* or *compensated* if at the switched on power supply the current in the diagonal of the bridge is absent.** It means, that potential difference $\varphi_c - \varphi_d = 0$. Hence, on shoulders of the bridge the potential difference and the current will be accordingly equal:

$$\begin{array}{ll} \varphi_a - \varphi_d = \varphi_a - \varphi_c; & I_1 = I_2 \\ \text{I)} & \text{II)} \\ \varphi_d - \varphi_b = \varphi_c - \varphi_b; & I_3 = I_4 \end{array} \qquad (6)$$

Applying the Ohm's law to each shoulder of the bridge, we shall receive:

$$\varphi_a - \varphi_d = I_3 R_T; \qquad \varphi_d - \varphi_b = I_4 R_M;$$
$$\varphi_a - \varphi_c = I_1 R_1; \qquad \varphi_c - \varphi_b = I_2 R_2;$$

Considering the first equality (6), we shall find:

$$I_3 R_T = I_1 R_1; \quad I_4 R_M = I_2 R_2.$$

Taking into account the second equality (6), we shall receive the ***working formula of the bridge circuit***:

$$R_T = R_M \frac{R_1}{R_2}. \qquad (7)$$

As resistors R_1 and R_2 are the homogeneous wire with the big specific resistance, that on the basis of the formula $R = \rho \frac{\ell}{S}$ the ratio of resistance R_1 and R_2 in (7) replace by the ratio of lengths ℓ_1 и ℓ_2. Then the formula (7) will become:

$$\boxed{R_T = R_M \frac{\ell_1}{\ell_2}}, \qquad (8)$$

where ℓ_1 и ℓ_2 are lengths of rheohord arms *bc* and *ac*.

Resistance of the box resistance R_M determine on numbers on the switches increased on the corresponding coefficient, marked on panel of the box.

Order of work

1. Research of dependence of resistance of thermistor from temperature (graduation of thermistor).

1. Place the thermometer and thermistor in the glass test tube which lower in the vessel with cold water ($\leq 15^0$ C).
2. Swicth on to the network the heater and the circuit. At temperature of 15^0C moving of the jockey of rheohord establish zero on galvanometer, write down lengths of shoulders ℓ_1 and ℓ_2 of rheohord.
3. Through everyone 5^0 C similarly write down values ℓ_1 and ℓ_2, heating thermistor up to 40^0 C.
4. Disconnect the heater. Cautiously get thermistor and the thermometer from the test tube for cooling within of 2-3 minutes. With help of (8) calculate R_T for each temperature.
5. Construct the graph $R_T=f(t)$, data bring in the table.

2. Measurement of the hand temperature.

1. Clamp the thermistor in the fist for one minute.

2. Establish zero on the galvanometer, write down lengths of shoulders of rheohord and calculate resistance of thermistor.

3. Under the graph find temperature of your hand.

t, ^{0}C	15	20	25	30	35	40
T, K						
l_1,cm						
l_2,cm						
R_T,Ohm						

4*. Under formulas (5) and (4) calculate width of the band gap of the semiconductor ΔE and its temperature coefficient of resistance α (under the instruction of the teacher).

Conclusion: with increase of temperature resistance of thermistor (semiconductor) decreases. Using the graph of graduation of thermistort is possible to determine unknown resistance or unknown temperature of biological object.

Control questions

1. Explain the formation of power zones of a solid.
2. Write down formulas of laws of change of resistance of conductors and semiconductors on temperature. Explain values included in them and units of their measurement.
3. Represent graphs R = f (t) for metals and semiconductors.
4. Define the thermal coefficient of resistance.
5. How does determine power width of the forbidden zone?
6. Draw the circuit of the bridge for determination of resistance of thermistor.
7. Deduce the working formula for determination of resistance of the semiconductor.
8. Application of thermistor in medicine.

LABORATORY WORK №22
STUDY OF THE WORK OF THE UHF-THERAPY APPARATUS

Purpose of the work: acquaintance with the principle of work of the UHF device, research of a spatial distribution of the electric field, construction of a resonant curve of the oscillatory contour, study of heating of substance by the field of UHF device.

Equipment: device UHF-66, dipole aerial, ammeter, two thermometers with the scale division value of 0.1^0C, vessels from the plexiglas, distilled water (dielectric), physiological solution (electrolyte), a stop watch.

Theory of the work

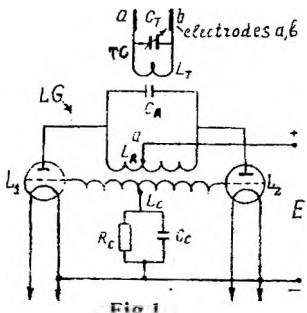


Fig.1

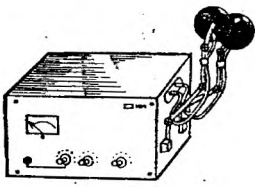
UHF-THERAPY is influence on tissues and bodies of *a variable electric field* of ultrahigh frequency (30 - 300 MHz). It is applied at treatment of inflammatory processes in bones and joints, for treatment of neuralgia, bronchial asthma and other diseases.

In this work is used the apparatus UHF-66 representing *the duple lamp generator with a capacitor feedback*. It works on frequency of 40.58 MHz. The device consists of duple lamp generator (LG) and therapeutic contour TC (see fig. 1).

Basic parts of LG:

1) anodal oscillatory contour L_AC_A in which not damped electromagnetic oscillations are supported;

2) two electronic lamps L_1 and L_2 (triodes) regulating supply of energy from the source to the contour two times for the period. From here is the name of the generator: duple;



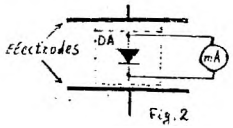


Fig.2

3) energy source E due to which in the contour not damped oscillations are supported,

4) feedback coil for maintenance of serial work of each lamp. The coil connects grids of both lamps. When on the grid of the lamp L_1 on the left end of the coil is "plus" than energy from the source acts in contour L_AC_A through the lamp L_1. In the second half of period lamp L_2 works similarly.

For influence by UHF electric field serves the *therapeutic contour* (TC) with two electrodes **a** and **b** (fig. 1). The therapeutic contour is connected with the contour L_AC_A inductively, that *excludes* an opportunity of *hit of the patient under the high voltage* available in LG.

The greatest power in the TC is allocated under condition of *resonance*, that is when frequency ω of oscillations in the contour $L_A C_A$ coincides with frequency of oscillations in the therapeutic contour: $\omega_A = \omega_C$.

Capacity of the TC is the sum of capacity of the variable condenser C_T and capacity between electrodes of patient C_p. At different procedures the capacity between electrodes of the patient changes, therefore **each time is necessary to make adjustment of TC in resonance, changing capacity of** C_T (the handle ADJUSTMENT). Control of adjustment is carried out with the help of the **neon bulb** (maximal brightness of the luminescence at the resonance) or with help of ammeter (max current).

The switch VOLTAGE adjusts work of the device in conditions of fluctuation of a voltage in the network. Control of the allowable voltage is made by pressing of the button CONTROL. The switch POWER changes the power of the generator (0, 20, 40, 70 Wt).

At work with UHF- generator is **FORBIDDEN**:

1) to switch on the apparatus without the permission of the teacher,

2) to replace electrodes, wires, safety locks, to connect grounding at the working device,

3) to bring to wires and electrodes metal subjects in order to prevent burns currents of high frequency. Metal subjects in UHF electric field are not heated, but near metal subjects there is concentration of power lines and energy (fig. 3e), that can lead to burns. Therefore from parts of a human body in the UHF electric field is necessary to remove metal subjects: rings, hours, hairpins, etc., and at presence of metal teeth, splinters in a body to show care, taking into account capacity of a field.

Distribution of intensity of the electric field between electrodes depends on their sizes and a relative positioning.

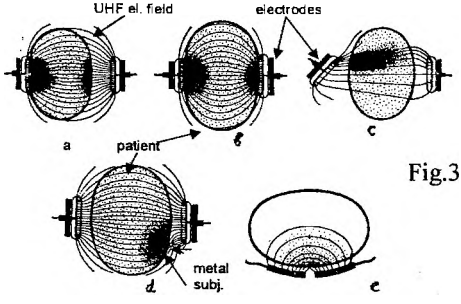


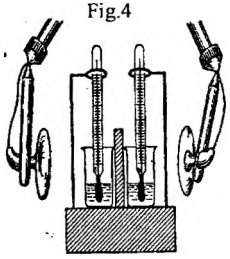
Fig.3

From figure 3 a,b is visible that the field between vertical parallel electrodes most homogeneously in the center and near the edges of electrodes power lines are strongly bent. Uniformity of the field is considerably broken at displacement of electrodes relatively of each other. Changing size of electrodes and its position concerning of a body is possible to provide some selectivity of absorption of UHF field energy. So, if electrodes are not identical more heat is allocated at a smaller electrode in the sizes.

Distribution of intensity of the UHF field we will study with help of dipole aerial (DA). DA is two conductors between which the diode (fig. 2) is included. DA is connected with the ammeter. The current arising in the contour of DA is proportional to intensity of UHF field. Moving DA between electrodes in the

horizontal plane is possible to study distribution of intensity of the electric field.

Fig.4



Effects of influence of UHF field on a tissues are divided on **not thermal** (change of physical and chemical properties of membranes) and **thermal** (deep heat on 5-7^0 C). On the big frequencies the capacitance of tissues (R=1 / (ω·C)) considerably decreases. About the same capacity gets also air, therefore UHF electric field freely passes air backlash between the electrode and a body and *will deeply penetrate into the tissues* inaccessible to other kinds of energy. This is *important advantage* of UHF method.

For study of thermal influence of UHF field on electrolytes and dielectrics between electrodes we establish vessels from organic glass with researched liquids. Temperature is measured by thermometers. Let's consider *mechanisms of action of UHF field* on solutions of electrolytes and dielectrics.

Heating *of electrolytes* in UHF field occurs due to *movement of ions* that is current of conductivity; thus energy of a current passes in internal heat. Quantity of heat allocated in electrolyte is: $q_{el} = \frac{E_{eff}^2}{\rho}$,

where E_{eff} is effective value of intensity, ρ is specific resistance of electrolyte.

In dielectrics under action of UHF field there are *rotations of dipole molecules*. Oscillations of dipoles lag behind on the phase oscillations of intensity of electric field. Quantity of heat allocated in dielectrics:

$$q_d = E_{eff}^2 \omega \varepsilon \varepsilon_0 tg\delta,$$

where ω is circular frequency, ε is relative dielectric permeability of dielectric, δ is an angle of dielectric losses.

Structure of human organism includes tissues possessing properties electrolytes and dielectrics, hence, in tissues the quantity of heat is:

$$q_\Sigma = q_{el} + q_d$$

As allocated heat depends on next values $\varepsilon, \rho, \omega$, that selecting the necessary frequency it is possible to make primary heating of the certain tissues and bodies. Because of various absorption of energy by albuminous molecules and ions *on frequency of 40.58 MHz heating of dielectrics (bones, nervous and fatty tissue, sinews and ligaments) occurs more intensively than electrolytes (blood, lymph, muscular tissues).* Except for that tissues-dielectrics are heated more strongly because of low heat conductivity and bad blood supply.

Order of work

1) Reception of resonant curves of the therapeutic contour.

a) Place the dipole aerial in the center between electrodes;

b) rotating the handle ADJUSTMENT through everyone of 10^0 (on the scale of the protractor), write down the indications of current I;

c) data bring in the table 1, construct the graph $I = f(\alpha)$.
d) Change distance between electrodes and repeat points a) and b).
Table 1

α, degree	0	20	40	60	...	180
I, mcA						

Conclusion: the form of a resonant curve depends on the position of electrodes and capacity of a site of a body located between electrodes.

2) Research of spatial distribution of the UHF field.

Table 2

Left		Right	
I, mcA	R, cm	R,cm	I,mcA
	0	0	
	1	1	
	2	2	
	3	3	
	4	4	
	5	5	
	6	6	

a) Establish the dipole aerial in the center between electrodes;
b) achieve a *resonance* and moving DA in the horizontal plane to the left and to the right from the center on 1 cm write down current I;
c) results of measurements bring in the table 2, construct the graph $I = f(R)$.
Conclusion: at motion from the center of electrodes to the device electric field intensity increases (bringing wires are source of UHF field).

3) Research of heating by UHF field of dielectrics and electrolytes.

a) Connect the of electrodes (at the switched - off voltage) with the vessels placed between electrodes and thermometers (fig. 4);
b) write down reference temperature of both liquids with accuracy of 0.1^0 C;
c) switch on UHF device and in the point close to the resonance through everyone 30 s write down indications of thermometers within of 3 minutes;
Table 3

t,s	t_{el},^{0}C	t_d,^{0}C
0		
30		
60		
90		
120		
150		
180		

d) results of measurements bring in the table 3, construct the graph of dependence of change of temperature of researched liquids on time.
Conclusion: on the given frequency tissues – dielectrics are heated better than electrolytes.

Control questions

1) What is the UHF-therapy? What is the working physical factor of the UHF-therapy?
2) What does represent the device of UHF-therapy? Basic parts of the device.
3) Purpose of the therapeutic contour. Why TC is adjusted in resonance with the anodal contour?
4) How we can show existence of resonance. Condition of the resonance.
5) On what factors does distribution of the field between electrodes depend?
6) Mechanisms of heating of electrolytes and dielectrics in the UHF field.
7) From what is depended the quantity of heat allocated in electrolytes and dielectrics? Write down formulas.

LABORATORY WORK №23

Determination of the refractive index of substance with the help of refractometer

Purpose of the work: to study the structure and principle of work of refractometer, to investigate dependence of the refractive index of sugar solution on its concentration.

Devices and accessories: refractometer, solutions of sugar of different concentrations, distilled water, cotton wool for cleaning of prisms of refractometer.

Theory of the work

The ray of light at transition through the border of two media changes the direction of its propagation. This phenomenon is known as ***refraction or refraction of light***. Let's consider laws of reflection and refraction of light.

Laws of light reflection:

1. The incident and reflected rays are in the same plane with the perpendicular that was lead to the border of two mediums in the point of incidence (fig. 1).

2. The incidence angle α is equal to the angle of reflection б, (fig. 1).

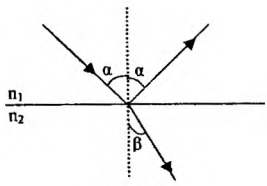


Fig. 1

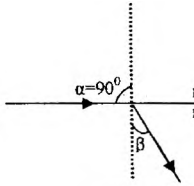


Fig. 2

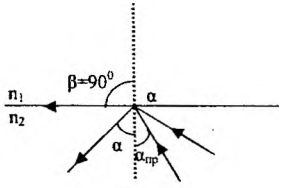


Fig. 3

Laws of light refraction:

1. The incident beam, the beam refracted and the perpendicular that was lead to the border of two mediums in the point of incidence, are in the same plane (fig. 1).

2. The ratio of the sine of incidence angle and the sine of angle of refraction is the constant for any two given media (Snell's law, fig. 1):

$$\frac{\sin\alpha}{\sin\beta} = \frac{\vartheta_1}{\vartheta_2} = n_{2,1}, \qquad (1)$$

where ϑ_1 and ϑ_2 are speeds of light in the first and second media, accordingly; $n_{2,1}$ is ***the refractive index*** of the second medium with respect to the first.

The ratio of speed of light in vacuum c to speed ϑ of its propagation in the given medium is called *absolute refractive index n* of medium: $n = \frac{c}{\vartheta}$. Then

$n_{2,1} = \frac{\vartheta_1}{\vartheta_2} = \frac{c \cdot n_2}{c \cdot n_1} = \frac{n_2}{n_1}$, i.e. the refractive index of two mediums is equal to the ratio of absolute refractive indexes of these mediums.

If light passes from the medium with the smaller refractive index to the medium with the big refractive index, than according to (1) the angle of incidence will be more angle of refraction. At falling of beam under the greatest possible angle ($\alpha = \frac{\pi}{2}$, i.e. the beam slides along border of media) the angle of refraction β_{cr} will be the greatest for the given mediums and it is called the ***critical angle*** (fig. 2).

Substituting $\alpha = 90^0$ and β_{cr} in the (1) we will receive: $\frac{\sin 90}{\sin \beta_{cr}} = \frac{1}{\sin \beta_{cr}} = \frac{n_2}{n_1}$ (2), whence: $\sin \beta_{cr} = \frac{n_1}{n_2}, n_1 = n_2 \sin \beta_{cr}$.

Using last expression is possible to find the refractive index of a researched liquid.

If the ray passes from the medium n_1 to the medium n_2 ($n_2 > n_1$) the angle of refraction will be more the angle of incidence. With increase of incidence angle the angle of refraction increases also until at some incidence angle $\alpha = \alpha_{cr}$ the angle of refraction appears equal to $\frac{\pi}{2}$ (fig. 3). The angle α_{cr} is the ***critical angle*** of total internal reflection. At incidence angles $\alpha > \alpha_{cr}$ all falling beams are completely reflected in the first medium ***(total internal reflection).***

Thus, the critical angle of total reflection for various substances in a solution *depends on their refractive index determined by concentration of a solution.* It has found application in refractometers (devices for measurement of a refractive index of substances). Refractometers are used at determination of the general albumin of blood serum, cleanliness of water and other substances, for identification of substances and determination of solubility.

Description of installation

Refractometer laboratory is intended for determination of a refractive index of a liquid within the limits of 1.3000 – 1.5400 (the scale of refractive index) and maintenance of dry substances on saccharose in products within the limits of 0-95 % (the scale of dry substances).

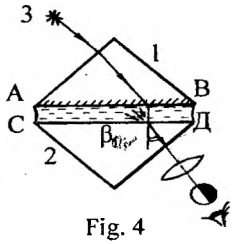


Fig. 4

The method of determination of the refractive index of a researched solution is put in the basis of the construction of the device *on the critical angle of refraction (for transparent liquids)* or on the angle of the total internal reflection (for the muddy, painted liquids).

The basic parts of refractometer are two rectangular prisms: 1 is lighting and 2 is measuring (fig. 4) prisms made of the same grade of glass.

Prisms adjoin their hypotenuses with the backlash of 0.1 mm. At determination of **n** for transparent liquid between prisms are placed some drops of liquid. A beam of light from the source 3 falls on the lateral side of the top prism and after refraction falls on the side AB. Surface AB is matt, therefore light dissipating and passing through the researched liquid falls on side СД of the bottom prism under various angles from 0 up to 90^0. If the refractive index of the researched liquid is less than refractive index of the prism, ray of light enters in the prism 2 under angles from 0 up to $\beta_{пр}$. The space inside this angle will be lightened and outside of it be dark (fig. 4). *Position of border of light and shadow is defined by the critical angle dependent on the refractive index of the researched liquid.* The optical scheme of the device is shown on fig. 5.

In our refractometer the source of white light is used (3). Because of dispersion at passage by light of prisms 1 and 2, border of light and shadow appears color. For elimination of spectral colouring before the objective 6 of visual tube is placed the equalizer 4. It consists of three prisms having different refractive indexes. Prisms are selected so, that the beam of $\lambda = 589.3$ nanometers (the wavelength of yellow line of Na) did not deviate. Beams with other wavelengths are deviated by prisms in the different directions. With the help of the special handle rotating the equalizer, we can remove colouring of border of light and shadow and to achieve its sharpness. Rays of light pass the equalizer and then gets in the objective glass of the visual tube 6. The image of border of light and shade is examined through the eyepiece 7. Simultaneously in the eyepiece is examined the scale 8 and the line of three strokes putting on the glass plate 9. The rotary prism 5 turns a beam on 90^0 for convenience of supervision.

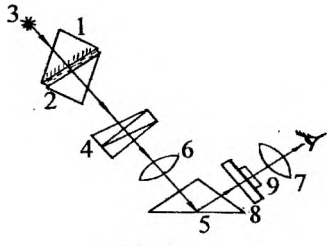


Fig.5

Order of work

Determination of the refractive index of the sugar solution and research of dependence of the refractive index on concentration of a solution.

1. Establish the light source 3 (fig. 6) so, that the top prism 1 was shined.
2. Turn down the top prism 1 and put by pipette 3-4 drops of water on the measuring prism 2. Smoothly lower the prism 1.
3. Rotating the eyepiece 5, achieve the precise image of the scale and of three strokes. Moving the eyepiece 5 by lever 6 find the border of light and shade. If the border is color, by the lever 4 of equalizer to remove colouring, to fix the lever by the screw 7 and rotating the lever 6 *precisely combine border of light and shade with three strokes.*

4. Write down the refractive index on the *left* scale (fig. 7). At correct adjustment

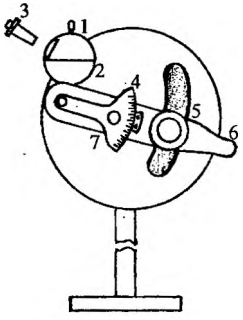


Fig. 6

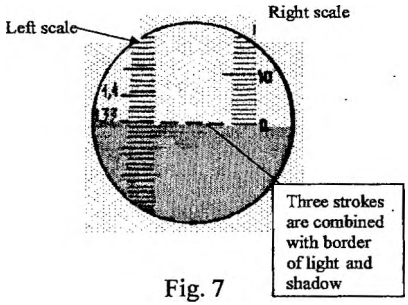


Fig. 7

of the device the dotted line for water coincides with division of scale 1.333. If indications will be others it is necessary to wash carefully the prism and again determine n for water. In case of difference of indications from 1.333 it is necessary to take into account a regular mistake in the subsequent measurements.

5. Having checked adjustment of the device, on the bottom prism put serially solutions of different concentrations and determines their refractive index. Results of measurements to bring in the table.

n								n_x
C %								

6. Construct the graph of dependence of the refractive index on concentration of the solution n = f (c).
7. Measure the refractive index of the solution of unknown concentration and with the help of graph find concentration of this solution.

Conclusion: refractive index of the sugar solution is directly proportional to concentration of the solution; using the graduated graph we can find unknown concentration of the solution.

Control questions

1. Formulate laws of reflection and refraction of light.
2. What is the absolute and relative refractive index?
3. What is the total internal reflection? What is the critical angle?
4. Using the law of refraction of light for the case of a critical angle, deduce the formula of the refractive index of the researched liquid.
5. Draw the optical circuit of the refractometer.
6. Why does measurement start from water?
   How does remove colouring of the border of light and shadow?
7. Application of refractometer in medicine.

LABORATORY WORK № 24

DETERMINATION OF CONCENTRATION OF SUGAR IN A SOLUTION WITH THE HELP OF POLARIMETER

Purpose of the work: to study principle of the work of polarimeter, to master the technique of determination of concentration of sugar in a solution with the help of polarimeter.

Devices and accessories: polarimeter, tubes with a solution of sugar of known and unknown concentration.

THEORY OF THE WORK

Light is the cross electromagnetic wave at which vectors of intensity of electric $\vec{E}$ and magnetic $\vec{H}$ fields are perpendicular to diretion of propagation of light and to each other. Chemical and *biological* action of light basically is connected with the *electric component* of light electromagnetic wave. In the natural light going from the Sun, a lamp, from the flame of a candle are superimposed the disordered radiations of set of chaotically radiated atoms, therefore fluctuations of light vectors $\vec{E}$ will not settle down in the same fixed direction and randomly directed in all directions, perpendicular to the beam (fig. 1).

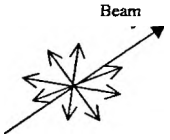


Fig. 1

Light which directions of oscillations of the vector $\vec{E}$ are regulated somehow is called **polarized**. If fluctuations of the light vector occur only in the certain plane, light is called ***plane polarized.*** The plane in which there are oscillations of the vector $\vec{E}$ is known as the plane of oscillations or the **plane of polarization** (fig. 2).

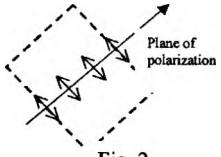


Fig. 2

Polarized light can be received at reflection or refraction of beams on the border of two mediums. However more often artificial polarization receives by passing of natural light through devices named *polarizers.* So receives polarized light using the phenomenon of **double refraction** of some crystals, for example, tourmaline and Icelandic spar.

The phenomenon of double refraction: the narrow monochromatic light beam falling on a surface of crystal is divided on two beams (**o** and **e**, fig. 3), which are taking place through the crystal on different directions. Intensity of each beam is equal to half of intensity of a falling beam. Both beams will be polarized in two mutually perpendicular planes.

At double refraction one of the beams satisfies to the usual law of refraction and is known as the *ordinary beam (o)*. For second beam named *extraordinary (e)* the law of refraction is not carried out at change of incidence angle.

The big application was received the polarizer named *prism of Nicol (or simply nicol).*

It represents a prism (rhomboid, fig. 3) from Icelandic spar (calcium carbonate), cutting on diagonal and sticking together by the Canadian balm (resin extracted from the Canadian fir). The refractive index of the Canadian balm n is between refractive index of the extraordinary n_e and ordinary n_0 beams ($n_0 > n > n_e$). It allows, having picked up in appropriate way angles of the prism to provide total internal reflection of the ordinary beam on the border with the Canadian balm.

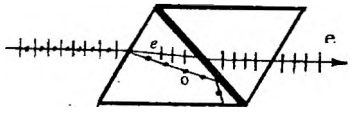


Fig. 3

The reflected beam in this case is absorbed by the black bottom side of the prism. The extraordinary beam passes through the layer of the Canadian balm and leaves from the nicol in parallel bottom side.

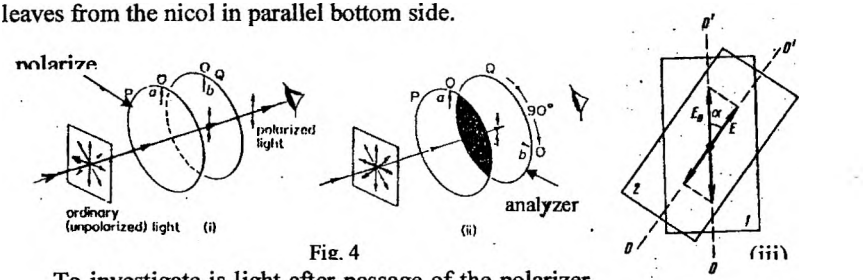


Fig. 4

To investigate is light after passage of the polarizer really polarized, on the way of beams put the second polarizer that is called the **analyzer** (on fig.4i and 4ii: P is polarizer, Q is analyzer). Let fluctuations of a vector $\vec{E}_0$ of the polarized light wave are made in the plane making the angle α with the main plane of the analyzer (fig.4iii). Then the amplitude of $\vec{E}_0$ can be spread out on two components: $E_\parallel = E_0 \cos\alpha$ conterminous to the main plane of the analyzer and $E_\perp = E_0 \sin\alpha$ perpendicular to it. The first component will pass through the analyzer; the second will be detained by it. As intensity of light is proportional to the square of amplitude of oscillations, we shall receive:

$$I = I_0 \cos^2\alpha, \qquad (1)$$

where I_0 is intensity of light incident on the analyzer, I is intensity of light which has left the analyzer, α is the angle between the main planes of the analyzer and polarizer.

The equation (1) expresses **Malus's law.** If planes of the polarizer and the analyzer are parallel ($\alpha = 0, \pi$; i.e. $\cos\alpha = \pm 1$) the screen placed behind the analyzer will be as much as possible alight. If $\alpha = \pi/2, 3\pi/2$, i.e. $\cos\alpha = 0$ (the polarizer and the analyzer are crossed) the screen will be dark.

At passage of polarized light through some substances is observed rotation of the plane of polarization. Such substances are called **optically active**. To optically active crystals concerns quartz, for example. Optical activity has not only crystals, but also liquids (turpentine, nicotine) and also solutions of some substances in water: solutions of saccharose , glucose, apple, almond acids; solution of camphor, strychnine in spirit.

In solutions the angle φ of rotation of plane of polarization is directly proportional to the way ℓ of the beam in the solution and concentration of the solution:

$$\boxed{\varphi = \alpha c \ell}, \qquad (2)$$

where *α is the specific rotation.* It is inversely proportional to the square of length of the wave λ, depends on nature of substance and its temperature, it is numerically equal to the angle of rotation of the plane of polarization by the layer of a solution by thickness of 1 dm at concentration of substance of 1 g per 100 cm^3 and at temperature of 20^0C. Specific rotation of sugar is equal to $66.5\frac{deg\,ree \cdot cm^3}{g \cdot dm} = 0.665\frac{deg\,ree \cdot m^2}{kg}$. There are substances which rotate the plane of polarization clockwise and counter-clockwise.

At passage of the polarized light through a solution of optically active substance, planes of polarization of waves of various lengths will rotate on different angles. Dependence of specific rotation on length of a wave is known as ***dispersion of optical activity***.

The parity (2) underlies on the basis of a sensitive method of measurement of concentration of the dissolved substances, in particular sugar. This method (**polarimetry, or saccharimetry**) is widely used in medicine for determination of sugar in urine, optical activity of serum fibers with the purpose of diagnostics, in biophysical researches and also in the food-processing industry. Corresponding measuring devices are called polarimeters or saccharimeters.

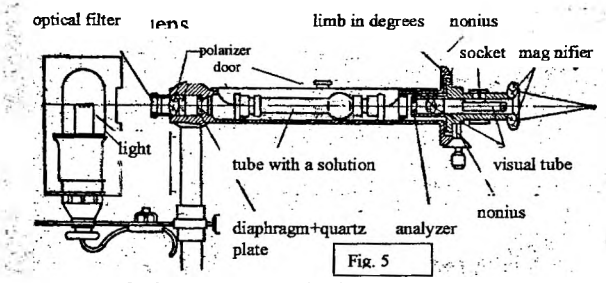


Fig. 5

Knowing specific rotation and length of a vessel with substance, it is possible to determine concentration of the solution under the formula (3):

$\boxed{c = \frac{\varphi}{\alpha \ell}}$ (3) – for concentration in grammas per 1 sm^3 of the solution;

$c = \frac{100\varphi}{\alpha \ell}$ - for concentration in grammas on 100 sm^3 of the solution.

Description of the device

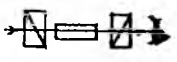


Fig. 6

Scheme of the elementary polarimeter is shown in the fig. 6. If between crossed (that is $\alpha = 90^0$ also will be the dark field of vision) polarizer P and the analyzer A to locate a tube with optically active material T, the field of vision will brighten up. Why? At transit of light through the tube the plane of its polarization rotates on some angle φ. If now the analyzer to rotate on the same angle φ the field of vision again becomes dark.

Knowing length of the tube, specific rotation of the solution and having measured φ, it is possible to find solution concentration **c**. However the measurement of angles and concentrations by such method is insufficiently precise as a human eye is *poorly sensitive to small changes of luminance of the field of vision.* But our eye is sensitive to the *slightest difference of luminosities of the next parts* of the field of vision. This property is used in the **half-shade** polarimeter. In our work the medical saccharimeter is used (fig. 4).

The optical scheme of the instrument is resulted on fig. 7.

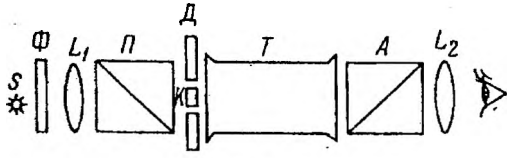


Fig. 7

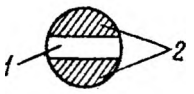


Fig. 8

Light from the lamp S falls on the filter F and objective L_1. Monochromatic light passes through polarizer П, vessel (tube) T with a solution and analyzer A. Then light passes through eyepiece L_2 of the visual tube of the saccharimeter that serves for visual supervision of the field of vision.

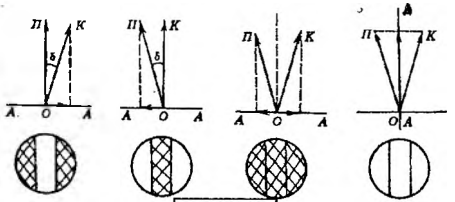


Fig. 9

Directly behind the polarizer diaphragm Д with a thin quartz plate K is located.

As quartz is optically active substance, after passage of polarized light through the plate its plane of polarization rotates on the some angle δ. As a result of it the field of vision of the saccharimeter is divided on three parts (fig. 8), which illumination depends on position of polarizer П and analyzer A. Uniform illumination (brightness) of the field of vision we can receive at two positions of the analyzer (fig. 9): 1) the plane of analyzer AA is perpendicular to bisector ПОК; 2) the plane of the analyzer coincides with bisector ПОК. In the first case brightness of the field of vision is less, than in the second. In this work it is necessary to equalize the field of vision on *smaller brightness*. If in established on equal illumination polarimeter to place the tube with a solution, equality of brightnesses of parts of the field of vision will be disturbed as planes of polarization in all parts of the field will be revolved on the same angle φ. For restoration of equality of illuminating intensities it is necessary to rotate the analyzer on the same angle φ.

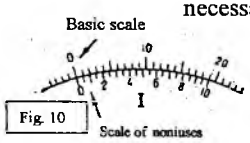


Fig. 10

Readout of angles is made on the limb of the basic scale which scale division value is equal to 1^0 and on two nonius, located at the left and on the right. The scale division value of the scale of noniuses at the majority polarimeters is equal to 0.05^0. The order of

readout is the following: 1) determine on how many integer divisions $n_{integer}$ is displaced zero of nonius in relation to zero of limb of the basic scale; 2) then determine number of divisions m from zero of nonius up to the division of nonius conterminous with division of limb and multiply this number on the scale division value (0.05^0) of nonius: $n^0=n_{integer}+m\cdot 0.05^0$ (fig. 10).

Order of work

1. Determination of specific rotation of sugar α

1. Switch on the lighter 4 of polarimeter (fig. 4).
2. Moving the coupling of eyepiece of the visual tube, establish eyepiece on clear vision of dividing lines of threefold field of vision.
3. Rotating the friction clutch to achieve uniform illumination of three parts of the field of vision. Thus door should be closed.
4. Find the readout φ_0 on the left nonius of the device. Measurement repeat three times and to find average value of $<\varphi_0>$.
5. Place the tube with the solution of sugar of known concentration **c** in the polarimeter. Thus identical illumination of the field of vision of the device is broken.
6. Rotating the friction clutch (clockwise) again achieve uniform illumination of three parts of the field, take off again readout on left nonius φ.
7. Measurements repeat three times and find $<\varphi>$.
8. Determine the angle of rotation of the plane of polarization $\Delta\varphi = <\varphi> - <\varphi_0>$.
9. Determine specific rotation of a solution of sugar: $\alpha = 100\Delta\varphi/\ell c$.
10. Results of measurements and calculations bring in the table 1:

Table 1

№ measurements	φ_0	φ	$<\varphi_0>$	$<\varphi>$	$\Delta\varphi$, degree	α, $\frac{deg\,ree\cdot cm^3}{g\cdot dm}$
1						
2						
3						

2. Determination of unknown concentration of a sugar solution

1. Place into the polarimeter the tube with a solution of sugar of unknown concentration and repeat operations 6-8 of task **1**, determine the angle of rotation of the plane of polarization for this solution $\Delta\varphi_x$.
2. Calculate unknown concentration c_x: $c_x = \frac{100\Delta\varphi_x}{\alpha_x \ell}$.
3. The data of measurements and calculations to bring in the table 2

Table 2

№ measurements	φ	<φ>	$\Delta\varphi_x$, degree	α, $\frac{degree \cdot cm^3}{g \cdot dm}$	c_x, %
1					
2					
3					

Control questions

1. What is natural and polarized light, ways of its reception?
2. What is the double refraction?
3. Course of beams in the prism of Nicol. Explain Malus' law.
4. What are the optically active substances? Give the examples.
5. Draw the optical scheme of the polarimeter.
6. Explain purpose of basic elements of polarimeter and the principle of its action.
7. Application of polarimeter in medicine.
8. How does determine specific rotation of a solution of sugar?
9. What is the dispersion of optical activity?
10. How does determine concentration of unknown solution of sugar?
11. Readout of angles of the polarimeter.

LABORATORY WORK № 25

STUDY OF PHOTOELECTROCOLORIMETER

Purpose of the work: to familiarize with the principle of operation and to determine optical density and concentration of the colored solutions.

Devices and accessories: photoelectrocolorimeter (ФЭК-М), galvanometer, set of solutions of the given substance with known concentration, solution of unknown concentration, set of cavities.

Theory of the work

Light intensity decreases at transition through a layer of substance. Reduction of intensity is result of interaction of light wave with electrons of substance. This phenomenon is known *as absorption of light.*

Let's establish the law of light absorption by substance. Let through homogeneous substance passes the bunch of collateral homogeneous rays of wave length λ. Let's pick out elementary layer of thickness dl (fig. 1). At transition of light through such field its intensity is decreased. The change of intensity *dI* is proportional to intensity I of incident light and thickness of the layer dl:

$$dI=-k_\lambda I dl, \qquad (1)$$

where k_λ is *the monochromic natural absorption index* depending on properties of the medium. The sign "minus" means that intensity of light decreases.

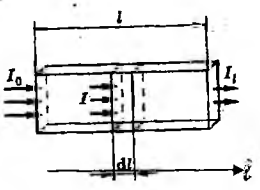


Fig. 1

Let's find intensity I_l of light past the layer of substance of thickness *l*, if intensity of light entered in the medium is equal to I_0. For this purpose we shall integrate expression (1), beforehand having separated variables:

$$\int_{I_0}^{I_l}\frac{dI}{I}=-k_\lambda\int_0^l dl, \qquad \text{in outcome we shall receive} \quad \ln I_l-\ln I_0=k_\lambda l$$

whence $$\boxed{I=I_0\cdot e^{-k_\lambda l}} \qquad (2)$$

It is **Bouguer's** *law (1729). It shows that intensity of light decreases in geometrical progression if depth of the layer will increase in arithmetical progression.* Graphically this dependence is expressed by the exponential curve (fig. 2). *The natural monochromic absorption index k_λ is value inversely proportional to distance l_e, on which intensity of light is decreased as result of absorption in the medium in e times* (fig. 2).

Light of the different wavelengths is absorbed by substance variously; therefore the natural monochromic absorption index k_λ depends on wave length.

*The monochromic natural absorption index of solution of absorptive material in not absorptive dissolvent is proportional to concentration "c" of a solution (**Bair's law**, 1852):*

$$k_\lambda = \chi' c, \qquad (3)$$

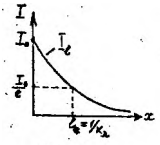


Fig. 2

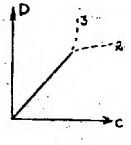


Fig. 3

where χ' is *natural molar absorption index.*

The Bair's law is fulfilled only for *weak* solutions. In strong solutions it is broken because of influence of interaction between close posed molecules of absorptive material. Substituting expression (3) in (2), we receive ***Bouguer — Lambert — Bair's law** (in the exponential form):*

$$I_l = I_0 \cdot e^{-\chi' c l} \quad \text{or} \quad I_l = I_0 10^{-\chi c l}, \qquad (4)$$

where $\chi \approx 0.43 \cdot \chi'$ is a *molar absorption index.* For monochromic light is used χ_λ a monochromic molar absorption index.

The ratio of $\tau = \frac{I_l}{I_0}$ is called ***transparency*** or *coefficient of permeability.* Transparency frequently express in percentage: for absolutely transparent solutions $\tau = 100\ \%$, for absolutely opaque $\tau = 0$.

Optical density of material is equal

$$D = \lg \frac{1}{\tau} = \lg \frac{I_0}{I_l} \qquad (5)$$

From expressions (4) and (5) we have $D = \chi c l$ (6)

It is Bouguer — Lambert — Bair's law (in more convenient logarithmic form): *optical density of solution (that is absorption of radiation) is proportional to concentration of absorptive particles and thickness of a layer of solution.* Graphically this dependence expresses by straight line (fig. 3, straight line 1). At deflection from monochromaticity of light, chemical modifications in a solution and change of refractive index the graph of dependence D=f (c) is deviated from straight line (fig. 3, curves 2 and 3). For absolutely transparent solution D=0, for absolutely opaque $D \rightarrow \infty$.

Bouguer – Lambert – Bair's law underlies of concentration colorimetry: photometric methods of determination of concentration of substance in colored solutions. In concentration colorimetry are used two groups of devices: objective *(photoelectrocolorimeter)* and subjective or visual *(photometers).*

Description of the unit

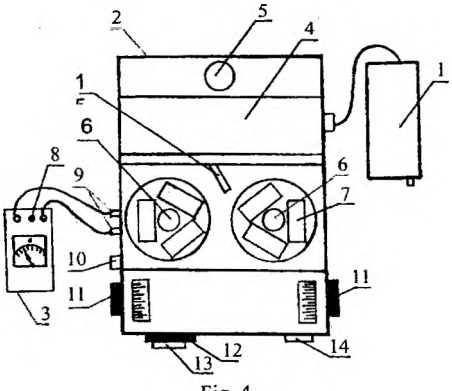


Fig. 4

Photoelectrocolorimeter is intended for determination of concentration of colored solutions. The device concerns to type of objectives, the principle of equalizing of intensity of 2 light bunches is in the basis of its work (with help of the slot-hole diaphragm).

The device consists of two parts (fig. 4): the stabilized power unit (1); the measuring block (2) with galvanometer (3).

On fig. 4 are submitted: the chamber (4) of color optical filters (neutral, green, dark blue, red); ventilating aperture for cooling the lighting lamp (5); holders of vessels (6); vessel (7); arrester of the galvanometer (8); jacks of connection of galvanometer (9); handle of the switch of sensitivity of galvanometer (10); the drum for change of the slot-hole diaphragm (11); the drum of the wedge of rough adjustment (12); the drum of the neutral wedge of exact adjustment (13); the handle of switching of color optical filters (14); the handle of oven-door overlapping light streams (15).

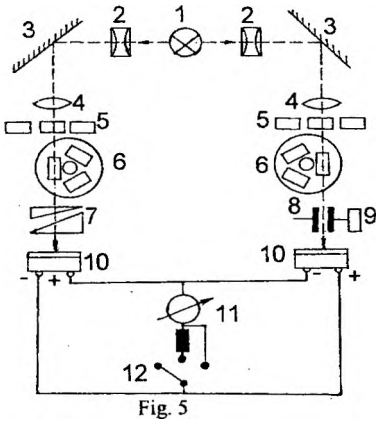


Fig. 5

The basic circuitry of the instrument is resulted on fig. 5.

Light beams from the lamp 1 through condensers 2, having reflected from mirrors 3, passes through objectives 4, optical filters 5, vessels 6 and gets on the photo cells 10, connected with the galvanometer 11 under differential circuit (at equality of intensity of light beams the arrow of galvanometer stands on zero). Change of sensitivity of galvanometer is carried out with the help of the switch 12. The slot-hole diaphragm 8 at rotation connected with it count drums 9 sitting on the same axis, changes the width and the value of the light stream incident upon the photo cell. Each drum has two scales: the scale of light-transmission factor (black) and the scale of optical density (red).

The unit of neutral photometric wedges 7 serves for weakening of light stream incident upon the left photo cell; the unit of color optical filters is intended for increase of accuracy of measurements.

Order of work

1. Preparation of the device for work.

1. To be convinced that galvanometer is connected with the device and the switch 10 (fig. 2) of galvanometer is in position "0".
2. Take off the arrester of galvanometer (the arrow on the button 8 should be ⊥ to the line connecting the plugs of the device) and if necessary to establish the arrow of galvanometer on "0" by mechanical proof-reader.
3. Switch on the power unit 1 in the network, put the toggle-switch in the position "on".

2. Choice of the optical filter.

1. Open the cover of device. If light beams are absent, with help of the handle 15 open oven-door. By the handle 14 of change of filters establish the neutral optical filter (H). To place on the way of the left and right light beams vessels of identical size with the 2 % solution. To close the cover.
2. Establish the index of right count drum 11 on zero division of the optical density scale (red scale).
3. Put the switch of galvanometer 10 in the position I. By rotation of the handle 12 of wedges of rough adjustment to establish the arrow of galvanometer on "0".
4. Put the switch 10 in the position II and by the handle 13 of wedges of exact adjustment establish the arrow of galvanometer on "0".
5. For the choice of the optical filter use two solutions of various concentrations. At determination of optical density, for example 8 % solution concerning of 2 % are necessary the vessel with the 2 % solution to place in the left holder of vessel. The same vessel with 8 % solution place in the right holder of vessel. Rotating of the right drum 11 to combine the arrow of galvanometer with zero. Put the switch 10 in the position II and again establish arrow on zero. On the red scale of the right drum determine optical density. The data bring in the table 1.

Table 1

Filter:	Neutral	Green	Dark blue	Red
D				

6. With the handle 14 to establish other optical filters and determine optical density similarly.
7. *For the further work is used filter for which the optical density of 8% solution appears* ***maximal.***

3. Determination of optical density of the colored solutions.

1. In the right and left bunch of light place vessels filled by water. Index of the right drum establish on "0" of scale of optical density (red scale).

2. The switch 10 of our galvanometer establish in the position I and rotating the drum 12 of wedges of rough adjustment establish the arrow of galvanometer on "0". The same to do at for the switch in position II by drum 13 of wedge of exact adjustment.
3. To place in the right bunch of light the vessel with a researched solution. Rotating the right measuring drum 11, establish the arrow on "0". The value of optical density D is counted on the right drum. The data place in the table 2.

4. Determination of unknown concentration of substance in a solution.

For determination of concentration of substance in a solution is necessary to construct a grade curve displaying dependence of optical density on concentration (D = f (C)).

Table 2

C%											
D											

Having measured optical density D of unknown solution under the graph D= f(c), determine its concentration.

Conclusion: *for weak concentrations the optical density of the solution is directly proportional to its concentration D=k·c.*

Control questions

1. Formulate Bouguer's law (with deduce and graph).
2. Formulate Bair's law.
3. Write down Bouguer – Lambert – Bair's law.
4. What is transparency of a solution?
5. What is optical density of a solution?
6. How does choose the optical filter?
7. How does determine the optical density of a solution?
8. How does determine unknown concentration of a solution?

LABORATORY WORK №26

FOCAL POWER DETERMINATION OF SPECTACLE LENSES BY MEANS OF DIOPTRIMETER

Purpose the work: to familiarize with the lens metre device (dioptrimeter), to determine with its help a focal power of spectacle lenses.

Devices and accessories: lens metre ДО-2, a set of spectacle lenses.

Theory of the work

The human eye is the complex optical system which has attained in the course of evolution high perfection. However in this system are proper both congenital and the acquired defects which are corrected by means of optical lenses.

Let's view the thin lens which focal distance is related to radiuses of curvature R_1 and R_2 its surfaces a relation:

$$\frac{1}{F} = D = (n-1)(\pm\frac{1}{R_1} \pm \frac{1}{R_2}), \qquad (1)$$

where $1/F = D$ a lens focal power, n is the refractive index of a lens material, sign «+» we take for a convex surface, sign «-» for concave. One diopter is a focal power of a lens with the focal distance of 1 m, i.e. 1 dp = 1/1m. The focal power of convex lenses is more than null, concave is less than null. In eye optics is used, mainly, convexo-concave lenses or meniscuses.

Spherical lenses are called stigmatic (from greek *stigma* – a point) since they shape the point image in space of images in the form of a point. The bundle of rays transiting through them, is termed as the homocentric.

Position of a back point of eye focus concerning the eye retina is termed a **clinical refraction**. If focal point is behind a retina the refraction is called *hypermetropic*, if on the retina is *emmetropic*, if ahead of the retina is called *myopic*.

The hypermetropia speaks: 1) the reduced optical force (focal power) of an eye, 2) the shorted shape of an eyeball. The myopia is consequence of 1) the incremented optical force of an eye, 2) the congenital extended shape of an eyeball or a consequence of both reasons.

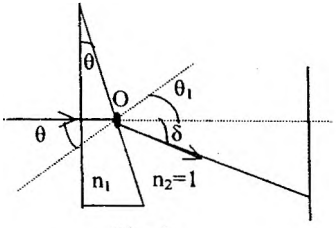


Fig. 1

Due to spherical activity of lenses, spectacles can correct sight at a myopia (negative lenses) and hypermetropia (positive lenses).

By viewing of a remote subject optical axes of both eyes are parallel also the eye works without an accomodation. For viewing of the close subjects the axis of both eyes should be crossed on observable object. For this purpose it

is necessary that the muscles twirling an eyeball functioned normally. If their function is broken, there will be a squint which can become further stationary values. For squint correction are applied prismatic lenses.

Prismatic lens (fig. 1) represents a thin glass prism with an apex angle θ and a deflecting angle δ: δ is the angle on which the incident ray is declined. For descriptive reasons a prism lateral view it is chosen in the form of a rectangular triangle and the incident ray is parallel the base. According to the law of refraction of light (for the point 0):

$$\frac{\sin\theta}{\sin(\theta_1+\delta)}=\frac{n_2}{n_1}. \qquad (2)$$

Considering that the prism is thin and all viewed angles are small, is possible to consider that sines of angles are approximately equal to angles (in radians) and since for air $n_2=1$ and (2) is equal:

$$\frac{\theta}{\theta_1+\delta}=\frac{1}{n_1}, \qquad (3)$$

taking into account equality $\theta_1=\theta$, (3) is equal

$$\delta=\theta(n_1-1). \qquad (4)$$

As for glass usually n_1 = 1.5 from (4) is visible that the deflecting angle δ of prism is ≈ equal half of apex angle. Therefore in glasses prisms with a deflecting angle more than 5^0 are used rarely: they will be very heavy. Deflecting activity of prisms (*prismatic activity*) is measured more often not in degrees but in prismatic diopters (Δ or pr. dp).

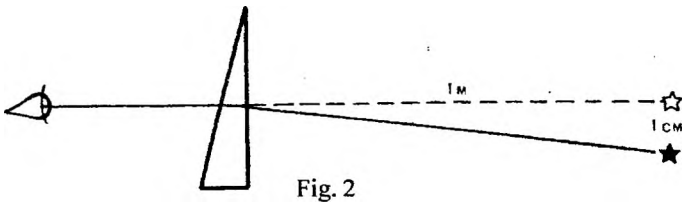


Fig. 2

The prism of force 1 prismatic diopter (1 Δ) declines the image of the subject that is on the distance of 1 m, on 1 cm towards vertex (fig. 2).

By definition a prismatic diopter: $1\Delta=10^2\frac{h}{\ell}=10^2\frac{0.01}{1}$. At use of prismatic lenses it is necessary to consider that at a converging squint (fig. 3a) the base of prism should be guided to the temple, at dispersing squint – to the nose (fig. 3б).

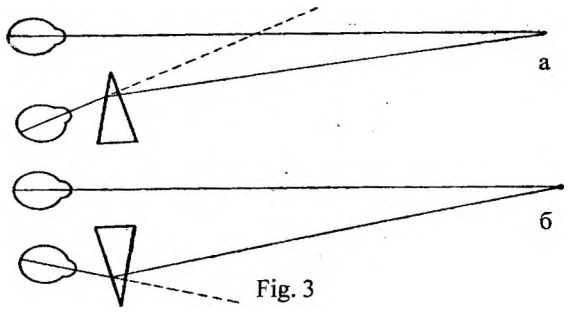


Fig. 3

Prismatic and spherical activity of lenses are interfaced. Thanks to prismatic activity the positive spherical lenses decline beams to centre and negative to periphery.

If the lens has unequal curvature in two different meridional planes, parallel incident rays will be crossed in the different points in relation to a refracting surface. The bundle of rays, forming such image is termed *astigmatic*, and a defect of this lens is known as **astigmatism**. The subject shape is thus garbled. The point is transmuted in the segment: the astigmatism means literally «point-free».

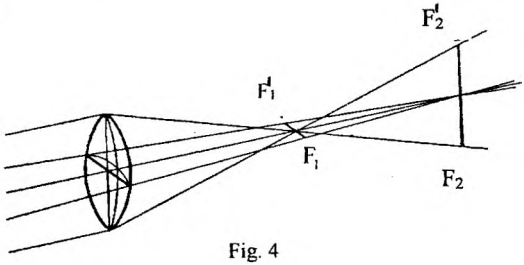


Fig. 4

Lenses with a cylindrical and torus surface have astigmatic activity. The lens with a torus surface is formed at gyration of a circle or its part round a straight line which is not transiting through its centre.

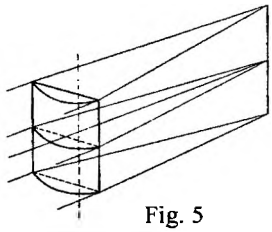


Fig. 5

The convex cylinder lens (fig. 5) collects a parallel beams in a line, parallel an axis of the cylinder and had in a focal plane of the lens. Cylindrical and torus lenses are called astigmatic.

Measure of astigmatism is the focal power difference in two main cross-sections. The the astigmatic difference is more, then it is more distance between focal lines F_1F_1' and F_2F_2'.

Astigmatic activity has any spherical lens if beams falls on it under a large angle to the optical axis (an astigmatism of skew fascicles). Astigmatism as optical defect of an eye is caused by disturbance of the spherical shape of a cornea and sometimes by irregular curvature of a lens. It shows, for example, in disability of

an eye to see sharply defined crossly perpendicular lines on the test table. The astigmatism of skew fascicles for an eye has no place as it is always plased in the direction of an observable subject.

If in one of meridians eye has the constant refraction the astigmatism is known as simple. It is eliminated by means of cylinder lenses. If in one meridian-section the refraction varies, then astigmatism is irregular.

Description of device

Lens metre ДО-2 (dioptrimeter) serves for measuring of a value of a focal power and prismatic activity of spectacle lenses.

Principle of work of the device is following (fig. 6). Lighted by means of a light source 1 and a condenser 2 lantern slide with the test mark 3 by means of the mobile lens 4 shapes the image on the screen 6. The measured lens is introduced into the space between mobile lens and the screen 5. To recover image sharpness, it is necessary to move the lens 4 forward or back. The number of divisions on a scale 7 on which the lens 4 is biased will specify a focal power of a measured lens.

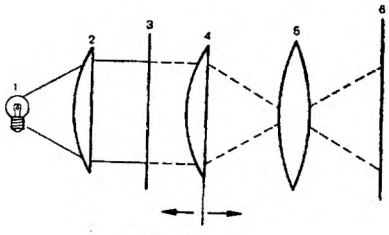


Fig. 6

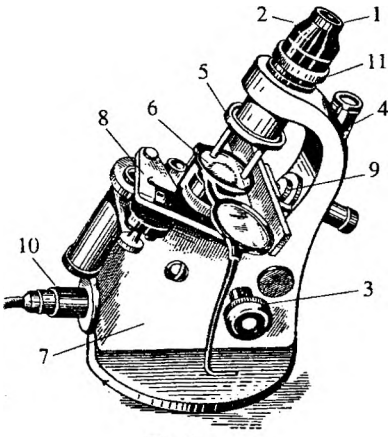


Fig.7

Lens metre ДО-2 figured on fig. 7 consists of following basic parts: collimator (it is a condenser and a lantern slide with the test mark), being in the case 7, optical tube 2, reading-off microscope 4, the gadget for hardening of glasses 5, mechanism for tagging-out of glasses 8, mechanism for measuring of diameter of glass 9, lighter 10.

Procedure of work

1. Preparation of the device to work.

1. Switch on the transformer and by rotation of the eyepiece carrier 1 of a tube 2 receive the sharp image of a grid with the cross-hairs.

2. Twirling a handwheel 3, receive a sharp image of a dot grid of "necklace" from luminous green points.

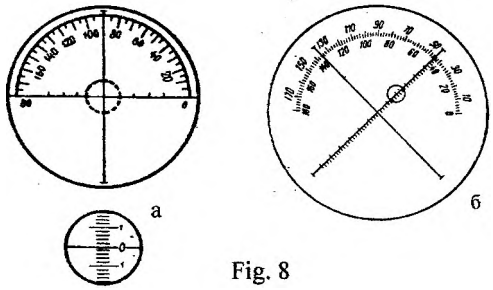


Fig. 8

3. By rotation of the eyepiece carrier of a reading-off microscope 4 receive the sharp image of a diopter scale. At the exact installation of a dot grid readout on a microscope should be equal to null (fig. 8a).

2. Sorting of lenses.

1. Establish glass on the device. For this purpose lift a pressure disk 5 with hinges and put the glass on the mobile sleeve 6. Slowly sink the pressure disk, having anchored by rods with rubber tips measured glass.

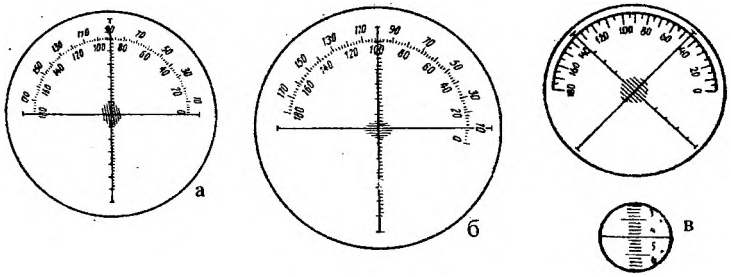


Fig. 9

2. By rotation of a handwheel 3 receive a sharp image of the luminous points. If points form a circle, the explored lens not astigmatic or prismatic.
3. For difference of these lenses from each other, it is necessary to uplift a pressure disk and cautiously move a measured lens. If the lens not astigmatic the dot grid is displaced and it can be easily established (fig. 8a) in the centre of cross-hairs, if the dot grid is immobile, a lens is prismatic. If a lens is astigmatic, instead of luminous points is seen a series of the parallel strips posed parallel to one of the main lens sections (fig. 9a).

3. Measuring of a focal power of not astigmatic lenses.

1. Establish a glass on the device and receive sharp image of the dot grid.
2. Place a dot grid in the centre of the cross-hairs.

3. On a scale of the reading-off microscope find indications of a focal power of a lens.

4. Determination of a focal power of prismatic glasses.

Taking into account, that the image of a dot grid of a prismatic lens places always outside of a cross-hairs, rotating a ring 11, establish the grid with a cross-hairs in such a way, that the scale with divisions transited through the centre of the dot grid (fig. 8б). The number of divisions digitized from the centre of the cross-hairs up to the centre of the dot grid, defines number of prismatic diopters.

5. Measurement of a focal power and astigmatic difference of astigmatic glasses.

1. Establish glass on the basic peg, by rotation of the handwheel 3 receive the clear image of parallel strips and bias them to a cross-hairs (fig. 9а).
2. Give the first scale reading of a reading-off microscope 4.
3. For measuring a refraction in the other section put group of parallel strips in a position, perpendicular first (fig. 9б). It is attained by slow gyration of the handwheel 3.
4. Make the second readout. It is recommended to give the first readout for section in which the refraction is less. The difference between first and second readout gives a value of astigmatism of a glass.
5. Standings of the main meridians can be designated on the degree semicircular scale with readout counter-clockwise (ТАБО scale). Arranging one of lines of the cross-hairs along strokes, on the dial find angles of the main meridians of a lens (fig. 9в).

At an assigning of astigmatic lenses after the number indicating focal power of a spherical element (**sph**) is put **cyl** (cyl – cylinder) also is underlined the sign and force of a cylindrical element in diopters, and also a standing of its axis *(ax)* on scale ТАБО. For example: sph – 0.5D, cyl – 1.0D, *ax* 10^0.

At execution of each step of the work all data put in the table:

№	Not astigmatic (spherical)	Prismatic	Astigmatic		
	Focal power, D, dp	Prismatic activity, Δ, pr.dp.	Focal power, D, dp		Astigmatic difference, dp
			1 merid.	2 merid.	

Control questions

1. Name the basic defects of vision. Specify the reasons of these defects.
2. What kinds of lenses are used for correction of vision?
3. Write down the formula of a thin lens.
4. What is the focal distance of a stigmatic lens of 5 dp?
5. Give the proof of the formula connecting a vertex angle of a prism with the deflecting angle.
6. What is the one prismatic diopter?

7. How is defined the measure of astigmatism?
8. The structure of the lens meter (dioptrimeter).
9. Determination by means of the dioptrimeter a focal power of stigmatic, prismatic and astigmatic lenses.

LABORATORY WORK №27

MEASUREMENT OF THE SIZES OF SMALL OBJECTS WITH HELP OF THE MICROSCOPE

Purpose of the work: determination of magnification of the microscope and its resolution, acquaintance with opportunities of its application for decision of practical problems.

Devices and accessories: biological microscope, photounit (camera adapter), object - micrometer, objects for researches.

Theory of the work

Microscope is the optical device intended for reception of increased images of small objects invisible to the naked eye.

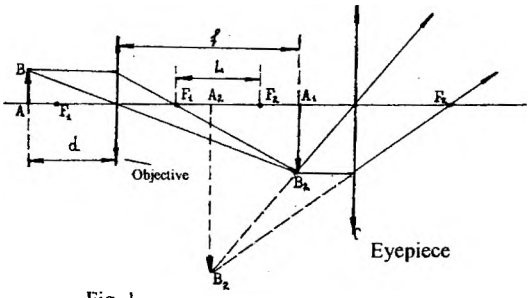


Fig. 1

The linear magnification of microscope N_M is defined by the ratio of the linear size of imaginary image A_2B_2 (fig. 1) to the true size of object AB. Apparently from the course of beams the magnification occurs twice: on objective and on eyepiece. Magnification of the objective is $n_{OB} = \frac{A_1B_1}{AB}$ and of the eyepiece is $n_{EYEP} = \frac{A_2B_2}{A_1B_1}$, then

$$N_M = n_{OB} \cdot n_{EUEP} = \frac{A_1B_1}{AB} \cdot \frac{A_2B_2}{A_1B_1} = \frac{A_2B_2}{AB}.$$

Taking into account, that at work the subject is placed near to focus of the objective ($F_1 \approx d$), and image A_1B_1 settles down at once behind forward focus of the eyepiece, it is possible to receive the following formula for **magnification of a microscope**: $N_M = \frac{L \cdot D}{F_1F_2}$, here L is *optical length of the tube*: distance between back focus of the objective and forward focus of the eyepiece ($f \approx L$); D = 25 cm is the least distance of distinct vision.

It is possible to assume that selecting values of L, F_1 and F_2 we can to achieve of very big magnification. However are actually limited to magnification in 1500-2000 times. It speaks the *phenomenon of diffraction at fine details* of considered object.

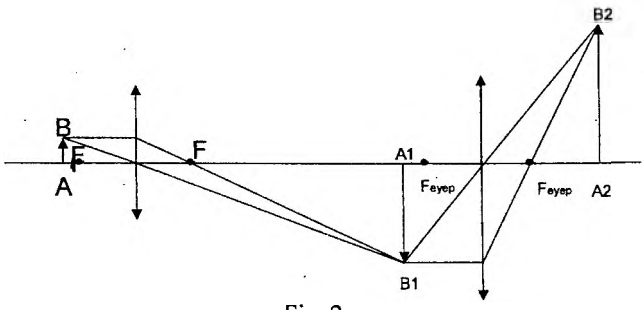


Fig. 2

The usual microscope itself does not give the valid image. However, if to displace tube of a microscope so that intermediate image A_1B_1 was before forward focus of an eyepiece (fig. 2), the beams leaving it will give the real image on the screen. This way of supervision on the screen is called *microprojection*. Photographing of the image received thus is called *microphoto*. For these purposes the special *photounit* is used, that is put on the eyepiece end of the tube of the microscope. Photographic plate is located on the place of the screen.

The linear magnification of photounit with a microscope is defined under the formula: $N_{UNIT} = n_{OB} \cdot n_{EYEP} \cdot \frac{x}{250}$, where x is the distance from the eyepiece of the microscope up to the photographic plate in mm; 250 is the least distance of distinct vision in mm. Value x for different photounits are various. For photounit with the size of 6.5×9 cm x = 140 mm. Then $N_{UNIT} = N_M \cdot 0.56$. Visual tube of our photounit has own magnification of 2.5. Therefore at supervision of object in the visual tube the magnification is equal: $N_{UNIT} = N_M \cdot 0.56 \cdot 2.5$.

The German scientist Abbe shows, that with help of microscope it is possible to see two points separately only if the distance between two points is

$$\ell_{min} \geq \frac{0.5\lambda}{n\sin(\frac{\alpha}{2})},$$

where λ is wavelength, n is the refractive index of the medium between the considered subject and the objective, α is *aperture angle*, i.e. the *angle between extreme beams of the conic light bunch which is included in the objective of the microscope*. Product $A = n \cdot \sin(\frac{\alpha}{2})$ is the ***numerical aperture*** of the objective. Numerical aperture is specified on the frame of the objective.

Value ℓ_{min} is known as the **distinction limit** of a microscope; it is the minimal distance between two points of the subject that in the microscope are

visible separately. Also it is used value inversely proportional to ℓ_{min}, that is called **resolving power:** $R=1/\ \ell_{min}$.

Average resolving power of a normal human eye on the distance of 25 cm is about $1.4\cdot10^4$ m^{-1} ($\ell_{min,eye}\approx0.075$ mm). In modern light microscopes this parameter is increased approximately in 400 times and finished up to $5.5\cdot10^6$ m^{-1} ($\ell_{min}\approx 2\cdot10^{-4}$ mm).

Taking into account presence of distinction limit of a microscope and resolving power of human eye, we enter concept of ***useful magnification*** of the microscope. If the subject has the size equal to the distinction limit of the microscope $\ell_{min,micr}$, that to increase the image is rationally till the size equal to the distinction limit of the human eye $\ell_{min,eye}$ on the distance of 25 cm. Then the *useful magnification* will be equal:

$$\Gamma=\frac{\ell_{min,eye}}{\ell_{min,micr}}=\frac{\ell_{min,eye}\cdot A}{0.61\cdot\lambda}.$$

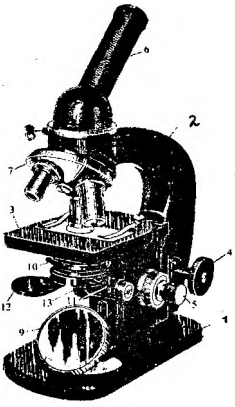


Fig. 3

It is considered that the eye is tired less, if the size of considered subjects (details of structure of object) in 2-4 times is more then $\ell_{min,eye}$. At illumination by light of λ = 0.55 microns human eye is most sensitive to this wavelength. Then values of useful magnifications are in the interval $500A<\Gamma<1000A$.

Description of device

In the microscope distinguish three basic parts: mechanical, lighting and optical (fig. 3).

Mechanical part consists of the basis 1; the holder of the tube 2 connected to the basis (the boot of the microscope); the subject little table 3; mechanism of rough focusing, that is actuated by the handle 4; the mechanism of thin focusing, that are actuated by the micrometric screw 5; the inclined tube (visual tube) of the microscope 6, in which the optical part is mounted; mechanism of change of objectives 7. The biological microscope has a demountable inclined tube that fastens by the screw 8.

The optical part consists of eyepieces and a set of objectives with different magnifications. The microscope is completed with replaceable eyepieces and objectives.

The lighting part consists from 1) mobile plane-concave mirror 9 used for direction of beams from the source of light on object (the flat mirror is used at work with immersion objectives); 2) condenser 10 and 3) diaphragms with the handle 11, that serves for creation of uniform and sufficient illumination of the field of vision; 4) demountable optical filter 12. Condenser moves up and down with help of the handle 13.

Demountable photounit МФН-1 (fig. 4) is intended for photographing of microobjects with the help of microcamera (the size of photographic plate is 6.5×9 cm). On figure 4 are shown the photounit and its parts. The basic collar 1 is put on the tube of the vertical unit 2 or on the ocular end of the microscope with vertical tube so, that the image of object is projected on the plane of the cartridge 3 of camera 4 and fixed on it by the screw 5. To the case 6 of the unit fastens the visual tube 7 inside which there is the objective and mobile eyepiece with the diopter mechanism 8. Between them the grid is located. For protection of eyes from strong light on the eyepiece the smoky optical filter 9 is put on. The unit is supplied by photoshutter, that is actuated by the trigger rope 11. Inside the case of photounit the reflective prism is located. Rotating the eyepiece for the frame of the tube of diopter mechanism we can achieve the sharp image of the grid and researched object. The same sharp image will be on the photographic plate also. Photounit has special handle, that allows to switch on or switch off the reflective prism to the course of beams going from object.

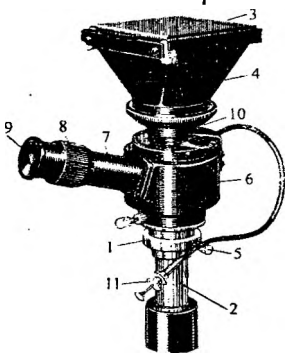


Fig.4

Order of the work

1. Determination of magnification of the microscope and the sizes of objects with help of photounit.

1. Establish the working objective (8×0.2 or 9×0.2, where 1-st number is magnification of the objective, 2-nd is its numerical aperture A).

2. Switch on the source of light, direct light on the mirror and close shutter of the camera turning the handle to the right. Turn the mirror and achieve bright illumination of the field of vision of the visual tube.

3. Maximum open the diaphragm of condenser and moving it, achieve uniform lighting of the field of vision.

4. Rotating diopter mechanism of the visual tube receive the precise image of its grid.

5. Place on the subject little table the object - micrometer and observing from the outside lower the tube of the microscope almost up to object. Then, observing into the eyepiece, slowly lift the tube and achieve the sharp image of the scale of the object - micrometer.

6. To switch the handle of the camera shutter to the left and to measure by the scale on the matte glass *length L of n integer divisions of the object – micrometer.*

7. Knowing the scale division value c (0.1 mm) of the object - micrometer (is rendered on the frame), calculate true length of n divisions: $L_{true} = c \cdot n$ (mm). Then the magnification of the microscope with the photounit will be equal: $N_{nozzle} = \frac{L}{L_{true}} = \frac{L}{cn}$ and without of photounit is: $N=N_{noz}/0.56$.

8. Place on the subject little table the cartridge with three wires of different diameter, receive their image on the matte glass and measure by the scale their increased diameters d_w.

9. Using value of N_{UNIT} determine true diameter of wire: $\boxed{d_{true} = \frac{d_w}{N_{UNIT}}}$.

10. Similarly find the diameter of the separate particles of lycopodium.

2. Determination of the sizes of objects with help of visual tube of the photounit.

a) Photoun it to switch on to the visual tube;

b) moving the object - micrometer on the subject little table to combine two scales: scale of ocular grid and of object - micrometer (fig. 5);

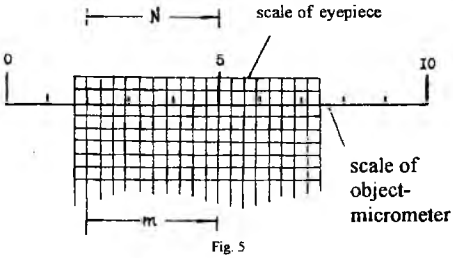


Fig. 5

c) if m divisions of ocular grid will coincide with N divisions of the object – micrometer, equality is carried out: $a \cdot m = c \cdot N$ and then the scale division value of the ocular grid is equal $\boxed{a = c \cdot N/m}$;

d) place serially on the subject little table each wire and determine their diameters under the formula: $d = a \cdot m$, where m is number of divisions of the scale of the grid on diameter of the wire;

e) similarly measure diameter of particles of lycopodium and to compare results to results of the 1 experiment.

3. Determination of resolution of the microscope.

Knowing the numerical aperture A (putted on the frame of objective) under the formula $\boxed{Z = 0.61\lambda/A}$ where λ = 555 nm, determine the distinction limit of the microscope Z and its resolving power: $\boxed{R = 1/Z}$.

Control questions

1. Represent the course of beams in the microscope and in the microscope with the photounit.
2. Deduce the formula of magnification of a microscope and write down the formula of magnification of a microscope with the photounit.
3. What is the distinction limit of a microscope and its resolving power?
4. Specify the ways of increase of resolving power of a microscope.
5. How does determine magnification of a microscope with the photounit?
6. How does determine the sizes of a subject with help of the photounit and the scale of its visual tube?

LABORATORY WORK №28

GRADUATION OF THE SPECTROSCOPE AND DETERMINATION OF WAVELENGTH OF SPECTRAL LINES

Purpose of the work: to learn course of beams of the spectroscope, to construct graduation curve, to study linear spectra of radiation of Hg and Na and absorption spectrum of $KMnO_4$.

Devices and accessories: spectroscope, spectral tubes, the table of wavelengths of spectral lines of gas in tubes (helium, mercury, hydrogen, etc.), light source (a lamp), spirit-lamp, test tube with solution of $KMnO_4$.

Theory of the work

According to the *first Bohr's postulate* there are stationary conditions of atom being in which it does not radiate energy (postulate of stationary conditions).

The *second Bohr's postulate*: the atom radiates (absorbs) quantum of electromagnetic energy at transition of electron from one stationary orbit to another:

$$E = h\nu = E_m - E_n .$$

At $E_m > E_n$ there is radiation of photon, at $E_m < E_n$ there is absorption of photon. Frequency of the photon arising or absorbed at transition is equal: $\nu = (E_m - E_n)/h$. The condition of atom in which it has energy more than in the basic condition is known as *excited.*

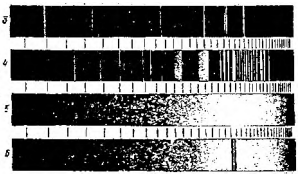


Fig.1

Spectrum is a set of the simple waves making the given complex radiation. The spectrum in which are available all waves gradually passing one in another is called **continuous** spectrum (fig.1, №5). Sources of the continuous spectrum are the heated solid and liquid bodies, gases at high pressure. The radiation consisting of several monochromatic waves gives a **linear** spectrum (fig. 1, №3). Source of a linear spectrum is radiation of gases and steams of metals in the atomic condition. For example, in the visible part of mercury spectrum steams the brightest lines are orange, yellow, green, blue and violet. If close located lines are united in strips the spectrum is called **band** spectrum (fig. 1, №4). Sources of a band spectrum are steams and gases in the molecular condition, for example, radiation of nitrogen.

Absorption spectrum corresponds to transitions of atoms from the basic to the excited condition. **Emission spectrum** corresponds to transitions from the excited condition to the basic. The spectrum of emission of atom is linear. Study of gases spectra has shown that to each of them corresponds certain linear spectrum.

Spectrum is source of different information.

By the *form* a spectrum is possible to identify atoms and molecules (*qualitative* spectral analysis). On *intensity* of spectral lines we can determine number of radiating (absorbing) atoms; it is the *quantitative* spectral analysis. For

the nuclear spectral analysis is used both spectra of emission and spectra of absorption. In the medical purposes the emission analysis is used for determination of microcells in tissues of human organism, small number of atoms of metals in tinned products with the hygienic purpose, some elements in cadaveric tissues for the purposes of forensic medicine.

The power spectrum of a free molecule is more complex than at atom. It consists of set of wide strips which represent closely located lines. Complexity of molecular spectra is caused by the big variety of motions in a molecule and, hence, power transitions.

The *molecular* spectrum is defined by presence in a molecule of motions of electrons, oscillatory motion of atoms about its position of balance and rotary motion of a molecule as whole and rotation of any part of molecules concerning others (internal rotation).

To these three kinds of motions corresponds three types of levels of energy: E_{el}, E_{osc}, E_{rot}. Total energy of a molecule is equal to its sum:

$$E = E_{el} + E_{osc} + E_{rot}.$$

Study of spectra of molecules gives data of the same character as study of atomic spectra. Molecular spectra of absorption are the important source of the information on biologically functional molecules and are widely used in modern biological and biophysical researches.

Description of the device

The elementary device allowing to carrying out the spectral analysis is the spectroscope (fig. 2).

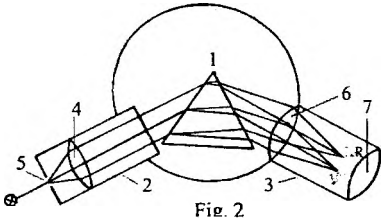


Fig. 2

The spectroscope consists of the prism 1, collimator 2 and visual tube 3. On one end of the pipe 2 there is the convex lens 4 located from the slit 5 on focal length. Beams of light are leaved collimator as parallel bunch.

Dispersion of light is carried out in the prism 1. Phenomenon of dependence of refractive index n of substance on frequency of light or from wavelength λ in vacuum is known as *dispersion* of light: $n = f(\nu) = \varphi(\lambda)$, where ν is frequency of a light wave.

Beams of the same color will leave the prism by parallel bunches and get to the visual tube 3. Lens 6 gathers beams of identical wavelengths in the focal plane forming a spectrum, which increased image is observed through the ocular lens 7.

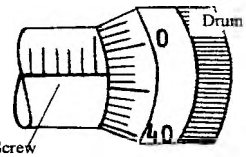


Fig. 3

For determination with the help of a spectroscope a wavelength of lines of researched spectrum it is necessary to ***graduate*** the spectroscope, *i.e. to establish dependence*

between wavelength λ of spectral lines and indications of the scale L of spectroscope reading-off device.

The scale division value of the micrometric screw 1 of reading-off device (fig. 3) is equal to 1mm and the scale division value of the drum (on which are 50 divisions) is equal: c=1mm / 50 =0.02 mm . Hence, *indications of reading-off device can be found under the formula* **L=N+nc,** where N is number of divisions (millimeters) counted on the scale of the screw, n is number of the divisions on the scale of the drum (*if n> 45 the stroke* on which costs the cut of the drum is *not taken into account*).

For *research of a linear spectrum* of any substance by means of a spectroscope receive the spectrum of this substance and write down in the table position of lines of the spectrum on the scale of spectroscope. For research of *absorption spectra* before the collimator it is necessary to put the source of light (continuous spectrum). In the eyepiece of spectroscope the continuous spectrum will be seen. Between the slit of collimator and the source of white light is located the researched substance, for example, the vessel with the solution of $KMnO_4$. Wavelengths which are absorbed by this substance will not be visible. Having written down position of dark lines (the line of absorption) on the scale of spectroscope with the help of graduate curve determines lengths of waves of absorption.

There is the catalogue of spectra of emission and absorption of all elements. Comparing the received spectrum with the tabulated λ it is possible to determine structure of substance (the qualitative analysis).

The quantitative spectral analysis is carried out by determination of intensity of spectral lines: intensity of line is proportional to quantity of a corresponding substance in the researched sample.

Order of work

1. Graduation of the spectroscope.

1. Familiarize with the device of the spectroscope and reading-off device.

Before the collimator instead of lamp we put the spectral pipe with gas with the known spectrum. For *steams of mercury wavelengths in nm* have the following values: red – 612.3, yellow – 579, green – 546, dark blue – 436, violet – 410 nm.

2. Determine position of all spectral lines on the scale of spectroscope and the data bring in the table:

Color	Length of the wave λ, nanometer	Position of an observable spectral line on the scale of spectroscope n, mm

3. Construct the graduation (calibration) curve of the spectroscope n= f (λ).

2. Research of emission spectra.

1. Establish before collimator the spirit-lamp which match is moistened in solution of NaCl. Establish the sawtooth ledge of spectroscope on the yellow line emitted by steams of Na and write down the indication of reading-off device.

2. By means of graduation curve determine the wavelength in the spectrum of the researched source of radiation.

3. Research of absorption spectra.

1. Between the lamp and the slit of collimator place researched substance (solution of $KMnO_4$).

2. On the scale of spectroscope determine position of the absorbed lines of spectrum. Data bring in the table:

Substance	Position of the absorbed sites of a spectrum on the scale of spectroscope n, mm	Lengths of waves of the spectrum absorbed by the given substance, nanometer

3. With the help of graduation curve determine the absorbed wavelengths (nanometer). Results bring in the table.

Control questions

1. What is the emission spectrum? Absorption spectrum?
2. Explain the origin of linear, continuous and band spectra.
3. What is the spectral analysis? Name its kinds.
4. Graduation of the spectroscope.
5. Basic parts of the spectroscope and the course of beams in it.
6. Determination of wavelength of a line in the spectrum of the substance.

LABORATORY WORK № 29

STUDY OF THE WORK OF THE GASEOUS OPTICAL MASER (LASER)

Purpose of the work: to familiarize with the arrangement and the principle of action of the gaseous optical maser; to find the divergence of laser ray, wave length of radiance with the help of diffraction grating; to study the principle of deriving of holographic images; to measure particle size of vegetable sulphur (licopodium), refractive index of glass.

Devices: the gaseous optical maser, diffraction grating, ruler, screen, optical bench, panel of holograms, vegetable sulphur, glass plate.

Theory of the work

Optical quantum generators (*laser* – Light Amplification by Stimulated Emission of Radiation) are based on generating and intensifying of light due to the forced induced radiance.

Induced radiance of electromagnetic waves originates, if the atom under action of the external photon passes from the excited state 2 (fig. 1) on the basic 1, radiating thus a new photon. Such interaction of the external photon with the excited atom occurs, if the energy hν of the photon is equal to the difference in level of energies of atom in the excited and basic states. In this case after interaction of our photon with the atom from atom already two photons will propagate, i.e. intensifying light is observed. The induced radiation formed at it has *the same frequency and phase*, as the radiance boosting this process.

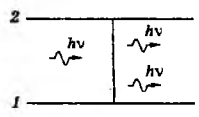


Fig. 1

Nonexcited atoms in substance is much more than excited, therefore at interaction of photons with matter dominates process of absorption and intensifying of light is not observed.

For prevalence of process of induced radiation above absorption is necessary to change distribution of atoms on energy levels. Intensifying of light will be observed, if concentration of atoms on the upper energy levels conforming to excited state is more than at the inferior levels. Such state is termed *as a population inversion.*

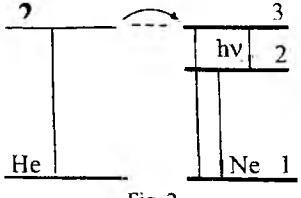


Fig. 2

The first quantum generators of electromagnetic waves were designed by Russian scientists N.G. Basov and A.M. Prokhorov and American physicist C. Townes (they have been awarded with the Nobel Prize in 1964).

Generators that give radiance in the optical range of wavelengths have received a name *of lasers.*

Let's consider principle of action and the arrangement (fig. 2 and 3) of *the gas helium-neon laser*. The laser necessarily has three basic parts: 1) the working medium in which states with inverted population are produced; 2) system of pump for creation inverse in the working medium; 3) the optical resonator excreting the selective direction of a fascicle of photons. The gas-discharge tube 1 is completed by the admixture of gases – helium and neon. Fractional pressure of helium is 1 mm. Hg and neon is 0.1 mm. Hg, in the tube number of atoms of neon in 10 times is less than atoms of helium. Atoms of neon are radiating (workers) atoms; atoms of helium are auxiliary, necessary for creation of inverted population of neon atoms. On the fig. 2 are shown energy levels of atoms of helium and neon.

At originating electric discharge in the tube, atoms of helium will excite and pass in the state 2. First excited level of He (2) coincides with energy level 3 of Ne atoms, therefore, colliding with atoms of neon, atoms of helium pitch them the energy and lead them in the excited state 3. Thus, in the tube the working medium consisting of atoms of neon with inverted population is created. Spontaneous transition of Ne atoms from energy level 3 on the level 2 creates radiance of photons. At the further interaction of these photons with excited atoms of neon there is the induced coherent radiation and in the tube there is increase of stream of photons. For augmentation of emissive power the tube is placed in the reflecting resonator consisting of two mirrors 5 and 6 (fig. 3). Being reflected from mirrors, stream of the photons flying along the axis of the tube, many times passes this tube. Thus process of induced radiance includes all greater number of atoms of neon; intensity of generated radiance will increase. The reinforced radiation stream through the mirror 5 goes outside. The gaseous optical maser works in *the continuous condition* of radiance. Because energy levels of 2 and 3 atoms of neon have the complex structure, the laser can radiate up to 30 *various wavelengths* in infrared and visual ranges. Mirrors 5 and 6 of resonator are made multilayer for creation owing to interference necessary reflection factor only for single wave. Thus, the laser gives *strictly monochromatic radiation*. For example: the helium-neon laser radiates wave length of 632.8 nanometers (red color).

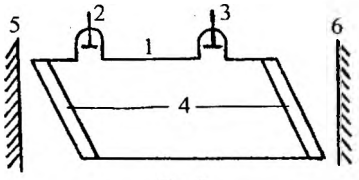


Fig. 3

The gas-discharge tube 1 from butts is obturated by plane-parallel glass plates 4 that are established under Brewster's angle to the axis of tube. Their position results in *plane polarization* of laser beams. For creation in a tube of electric discharge, in it two electrodes are built: the anode 2 and the cathode 3 to which high voltage (1.5-2 κV) is brought.

The laser radiation has the following *properties*: 1) temporal and *spatial coherence* (compounded passing in time and space of several wave processes), *2) severe monochromaticity, 3) polarization, 4) small angle of divergence, 5) big*

energy of stream density. On these properties is based application of lasers. Now lasers are widely used in various fields of medicine.

For the first time the laser has been applied in *ophthalmology* for treatment of layer separation of the retina. Laser beams are applied and to treatment of some initial forms of intraocular tumors.

In *surgery* the laser beam allows by absolutely sterile "light scalpel" to dissect tissues and perform operation almost without bleedings as small and average vessels are soldered. Destructive action of a laser ray is used for treatment of pigment nevuses, warts and tumors.

Application of nonrigid optical guides allows use laser light for intrinsic coagulation and deriving of holograms of internals, for example, the stomach.

High coherency of laser light has allowed realize essentially new photographic technique: deriving of a stereoscopic picture or *holograms*. In a routine photo the image is received two dimensional. The stereoscopic picture has been called as the hologram, and a method – holography (Hungarian D. Gabor, 1947, the Nobel Prize, 1971).

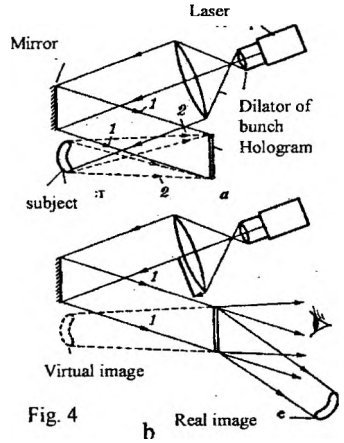


Fig. 4

In the figure 4 a is shown the installation diagram for deriving of holograms, and in figure 4 b is given the refresh circuit of the plotting. The light bunch irradiated by the laser dilates with the help of system of lenses and then is divided on two parts. First part is reflected by the mirror to the photoplate, forming so-called *basic* bunch 1. The second part hits on the photoplate, having reflected from the photographed subject; it forms *the object* bunch 2. Basic and object waves are coherent and overlaps against each other, forming on the photoplate interference pattern. After exhibiting a photoplate is received *the hologram: the interference pattern registered on a photoplate.*

For image reconstruction the hologram is placed in the same position, as well as at photographing. The hologram illuminates the basic bunch of the same laser (the second part of the bunch is blocked by the diaphragm). As result of diffraction of light on interferential structure of the hologram the duplicate of the object wave is recovered, generating volumetric virtual image of the subject, located in the place where the subject was at photographing. Besides the real image of the subject is formed the virtual image. Routinely is used the virtual image which creates the complete illusion of existence of substantial subject. If the hologram to split on some pieces each of them recovers the total image. However decrease of the dimensions of the hologram reduces image sharpness. It speaks that the hologram serves as a diffraction grating, and at decrease of number of slits its resolution decreases.

Description of installation

The laser (1) is positioned on the optical bench on fixed distance from the screen (3) (fig. 5). Near to the output window of the laser on a support the diffraction grating (2) is positioned. On the screen (Э) it is possible to observe a diffraction pattern (fig. 6). Wave length of the laser is determined by formula: $d \sin \alpha = k\lambda$,

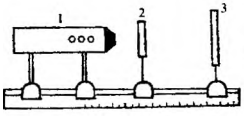


Fig.5

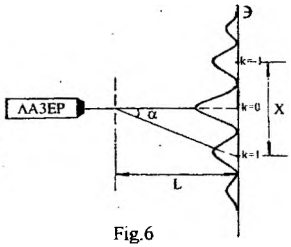


Fig.6

where d is grating constant (period); k is order of the maximum and α is the angle. The sine of the angle can be found from the formula (for vanishing angles $\alpha < 0.08$ radian): $\sin\alpha \approx \mathrm{tg}\alpha = \frac{X}{2L}$, where L is distance between the grating and the screen; X is distance between maximums of the same order. Then $\lambda = \frac{d \cdot X}{2L \cdot k}$ (1)

For determination of a divergence (angle apostatis) of laser ray, changing position of the screen measure diameters D_1 and D_2 of laser stain at two positions 1 and 2 of screen and distance L (fig. 7). By formula

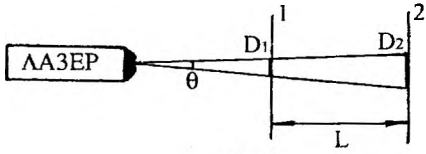


Fig. 7

$\theta \approx \mathrm{tg}\theta = \frac{D_2 - D_1}{L}$(Radian) (2) find the divergence θ of the beam.

If in the capacity of the diffraction grating to use the monolayer of small particles of the equal dimension (fig. 8), located chaotically on the screen, it is possible to observe a diffraction pattern. The pattern represents the sum of diffraction pattern from separate particles as concentric alternating dark and light rings. At the center there is a light circle.

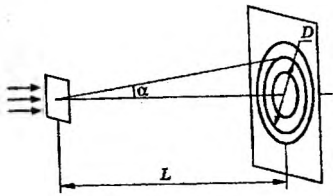


Fig. 8

From the diffraction theory of Huygens - Fresnel follows that at diffraction of collateral rays on a round barrier dark rings are received under the following conditions:

$\sin \alpha_1 = 0.61\ \lambda/r$, $\sin \alpha_3 = 1.11\ \lambda/r$, $\sin \alpha_5 = 1.62\ \lambda/r$, (3)

where λ is wave length of light, r is radius of the barrier, α is angle radius of the ring.

Requirement of deriving of light rings: $\sin \alpha_2 = 0.82\ \lambda/r$, $\sin \alpha_4 = 1.34\ \lambda/r$. (4)

Thus, using the diffraction pattern is possible to determine particle sizes on which there is diffraction:

$$r = \frac{m\lambda}{\sin\alpha}, \qquad (5)$$

where m is the order number of the ring. Rings number from the first dark ring (m=0.61). Angle α determine from the formula

$$\text{tg}\,\alpha = D/(2L). \qquad (6)$$

where D is diameter of the ring. As diffraction rings are received wide

$$D = (D_1+D_2)/2, \qquad (7)$$

where D_1 and D_2 are external and internal diameters of ring.

Laser refractometry. One of the most sensitive methods of determination of refraction of an eye is the method based on neutralization of seeming motion of objects: a method of laser refractometry. What is the eye refraction? The *physical refraction* is a refracting force of optical system of a human eye (≈ 63 dioptres). The *clinical refraction* of an eye characterises position of the main focus of optical system of the eye. If the focus point lies behind the retina refraction is *hypermetropic* H, if it is on a retina – refraction is *emmetropic* Em, if ahead of the retina refraction is myopic M. In practical activity the ophthalmologist defines only the clinical refraction that defines not only by the form of refraction (a far-sightedness or short-sightedness), but also on degree. Refraction degree is measured by optical force of the lens corrective a defect of vision, for example: the refraction at myopia can be equal to (– 3) dioptres.

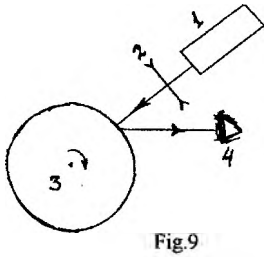


Fig.9

The method of laser refractometry is based on the phenomenon of *interference of coherent rays* of light in the human eye. Light from a coherent source (laser) is reflected from rough (for example, a metal) surface and getting to the eye forms on the retina a characteristic pattern: non-uniform light exposure, so-called, laser granularity, "shagreen leather" (speckle interferogram). Thus, if the eye and a reflecting surface are moved relatively to each other this picture is represented to the observer also moving. The direction of this motion depends on refraction of the investigated eye: for the mope-eyed eye the pattern moves to the same direction as the reflecting surface. For a far-sighted eye the pattern moves in the opposite direction, for the emmetropic eye (normal vision) pattern not moves, as though "boils". The reason of such motion is clear from fig. 10: the beams reflected from the points A and B of the screen get on the retina of a short-sighted eye in the points A_1 and B_1, and on the retina of far-sighted eye in the points A_2 and B_2. At myopia points A_1 and B_1 moves to the same direction as reflecting screen, and at a

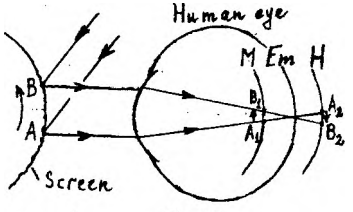


Fig.10

far-sightedness the points A_2 and B_2 moves aside opposite to screen movement (and the focus point O of emmetropic eye remains motionless). Compensating by positive or negative lenses motion of speckle it is possible to determine the eye refraction with high accuracy. The laser refractometry is applied for rough definition of refraction at mass screenings, or at determination of a spherical component at correction. Sensitivity of the method is not worse than 0.125 dioptries.

Installation for demonstration of the laser refractometry (fig. 9) consists of a source of laser radiation 1, a concave lens 2 doing a laser beam wide and the rotating drum 3 with a metal surface (kymograph), reflecting a laser beam. Observer 4 is on the distance of 2-3 m from the drum of kymograph.

Procedure of the work

1. Determination of wave length of radiation of the gaseous laser.

1. Establish the diffraction grating perpendicularly to axes of the laser. Switch on the laser, observe the maximums of the 3-d order.
2. Measure by a ruler distance L (fig. 5) between the grating and the screen and distance X between first-order maximums.
3. By formula (1) determine wave length λ_1 of laser radiation.
4. Lead analogous measurement and evaluations for second-order maximums and define λ_2. Calculate average value of $\langle\lambda\rangle$.
5. Observed data and results of calculations bring in the table.

№	X, cm	L, cm	d, mm	λ_1, nm	λ_2, nm	$\langle\lambda\rangle$, nm
1						
2						

2. Determination of the laser beam divergence.

1. Receive a stain of the laser on the removed screen and measure its diameter D_2 (fig. 7).
2. Receive the stain on the screen located closer to the laser and measure the diameter of the second stain D_1.
3. Measure distance L (fig. 7) between two positions of the screen.
4. By formula (2) determine the divergence of laser beam.

3. Particle-size determination of licopodium (spore of seeds of a club moss).

1. Establish on the optical bench the frame with licopodium and receive a legible diffraction pattern (fig. 8).
2. Measure distance L between the frame and the screen.
3. Measure external and internal diameters of the first dark ring and calculate the diameter of the ring D by formula 7.
4. Calculate $\mathrm{tg}\,\alpha_1$ by formula 6 and under the table find $\sin\alpha_1$.
5. Calculate the size of a particle of licopodium by formula 5.

6. Do analogous measurements for the following dark and light rings and calculate average value of <r>.

Outcomes bring in the table.

The number of diffraction ring	m	L, mm	D_1, mm	D_2, mm	D, mm	tgα	sinα	r, mm	<r>, mm
1									
2									
3									

4. Determination of the refractive index of the glass plate.

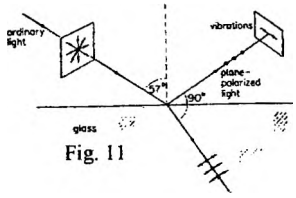


Fig. 11

At incidence of natural light on a demarcation of two dielectrics the reflected and refracted rays are in part polarized (fig. 11). If the angle of incidence i_B fulfils to a requirement (Brewster's law)

$$\tan i_B = n, \qquad (8)$$

(where $n=n_2/n_1$ is relative refractive index of two mediums) the reflected ray is completely polarized in the plane of incidence and in the refracted ray oscillations collateral to the plane of incidence prevail. If in a laser beam incident upon Brewster's angle oscillations of the vector $\vec{E}$ are carried out only in the plane of incidence the reflected ray will miss (I=0).

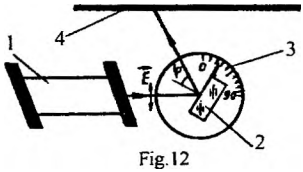


Fig.12

1. Establish the glass plate (2) it agrees fig.12. Switch on the laser (1) and slowly turning the plate concerning the scale of graduated arc (3), observe the stain of reflected light on the screen (4) (or on the ceiling) and change of its intensity. At the moment of disappearance of the stain determine on the graduated arc the angle i_B.

2. Iterate item 1 3 times, define average value of angle and by formula 8 calculate refractive index of the plate.

5. Observation of a holographic image.

On the path of sunlight or laser beam locate the hologram from the panel. Observe holograms of the mechanism of clocks (white casing in the panel), orbicular mirror (red casing), volumetric phase diffraction grating (yellow), etc.

6. Determination of character and degree of refraction of a human eye.

Closing each eye by turns observe without straining of eye a laser picture (speckle) on the drum of kymograph. In a direction of motion of the speckle draw a conclusion on character of refraction of each eye. Using the lenses from a "small set trial lenses" define degree of refraction of your eyes.

Control questions

1. Describe the mechanism of originating of induced radiance.
2. Specify key properties of induced radiance.
3. What state in atom is termed inverse?
4. Describe the arrangement and principle of the gaseous optical maser action.
5. Key properties of laser radiation.
6. How inverted population of atoms in the helium-neon laser is created? What is assigning of mirrors in the gaseous optical maser?
7. How the divergence of laser is determined?
8. Determination with the help of the diffraction grating of laser wavelength.
9. Determination of the particle size of licopodium.
10. Application of lasers in medicine.
11. What is hologram? Scheme of reception and observation.
12. What is the basic idea of determination of refraction of the human eye by the method of laser refractometry?

LBORATORY WORK № 30
USE AND APPLICATION OF THE DOSIMETER-RADIOMETER АНРИ–01–02 "PINE (COCHA)" FOR RADIATION CONTROL

Purpose of the work: to familiarize with a procedure of use of the device for radiation measurements.

Devices and accessories: dosimeter-radiometer АНРИ–01–02 "PINE", radioactive preparations in the lead housings, three basins from a device complete set, set of materials for examination (glass plate, board, sand, ashes and water), stop watch, support.

Theory of the work

Property of unstable nucleus of some elements spontaneously (i.e. without any influences) to transform to nucleus of other elements with emission of ionizing radiation is called **radio-activity**. The phenomenon is known as **radioactive decay**, it is a *statistical phenomenon.*

Let for small time interval dt decays dN nucleus. This number is proportional to the interval of time dt and also to the general number of the radioactive nucleus, which have not decay yet to the beginning of the given time interval:

$$dN = -\lambda N dt, \qquad (1)$$

where λ is *decay constant* (characterizes probability of decay of a nucleus per unit of time and it is various for various radioactive nucleus). Dimension of decay constant is s^{-1}. The mark minus specifies decrease in time of value N, i.e. $dN < 0$. Expression (1) represents the differential equation of the 1-st order with divided variables. We shall divide variables and we shall integrate in view of that the bottom limits of integration correspond to entry conditions: at $t=0$, $N=N_0$, where N_0 is initial number of radioactive nucleus:

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt;\ \ln\frac{N}{N_0} = -\lambda t;\ \ln\frac{N}{N_0} = \ln e^{-\lambda t};\ \frac{N}{N_0} = e^{-\lambda t};$$

$$\boxed{N = N_0 \cdot e^{-\lambda t}}, \qquad (2)$$

i.e. number of radioactive nucleus, which have not broken up yet decrease on the exponent law. Expression (2) also is the **basic law of radioactive decay.**

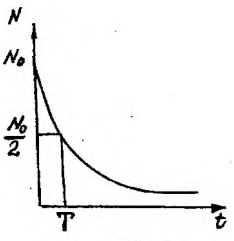


Fig. 1

Rate of decay of various radioactive elements characterize *half-life period* T. T is time during which half of initial number of radioactive nucleus (fig. 1) decays. We shall establish relationship between T and λ. The half-life period can be determined from the following reasons: at $t = T$;

$$N = \frac{N_0}{2};\ \frac{N_0}{2} = N_0 \cdot e^{-\lambda T};\ \frac{1}{2} = e^{-\lambda T};\ \ 2 = e^{\lambda T};\ \ \ln 2 = \lambda T;$$

$$0{,}693 = \lambda T;\ \ T = \frac{0{,}693}{\lambda}.$$

In conditions, when radioactive radiation is used for any purposes (for example, in medicine) it is necessary to know total number of decays per unit of time in the given quantity (mass) of radioactive element. This value is rate of decay and is known as **activity** (A). It is the essential characteristic of a radioactive preparation: $A = -\frac{dN}{dt}$, since $-\frac{dN}{dt} = \lambda N$ and $N = N_0 \cdot e^{-\lambda t}$, then

$\boxed{A = \lambda N_0 \cdot e^{-\lambda t}}$. Initial activity (t=0) $A_0 = \lambda N_0$. Then $A = A_0 \cdot e^{-\lambda t} = A_0 \cdot e^{\frac{-0{,}693}{T}t}$.

The activity calculated for a mass unit of an isotope is called *specific activity*. For solutions as specific activity understand activity of radioactive solution of 1 ml.

There are several types of ionising radiation:

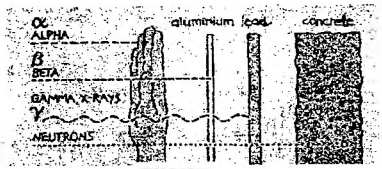


Fig. 2

Unit of activity in SI is 1 Becquerrel (Bc), that corresponds to activity of a radioactive source in which for 1 second there is 1 act of decay.

The most used unit is *Curie* (Cur):

$$1\text{Cur} = 3.7 \cdot 10^{10}\ \text{Bc} = 3.7 \cdot 10^{10}\ \text{s}^{-1}.$$

Ways of protection against radiation are various: *for protection from α- particles there is enough layer of a paper, clothes; from β- radiation it is possible to be protected by centimetric layer of a tree, glass or any easy metal; for protection from γ- radiation are applied thick (up to meter) layers of water, concrete, brick walls and also plates of lead by thickness up to 10 cm (fig. 2).*

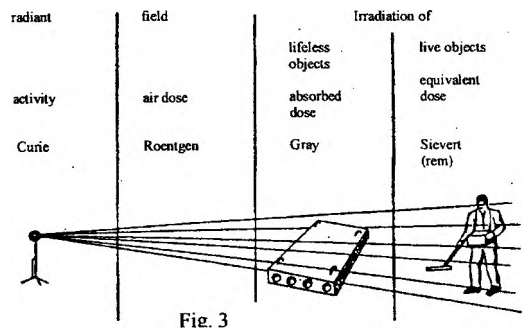


Fig. 3

Physical action of an ionizing radiation on a tissue of human organism consists in processes of excitation and ionization of atoms and molecules. Chemical bonds of molecules are thus opening and molecules disintegrate on composite chemical radicals. It causes disturbances in a

normal operation of the cell and can lead to its loss.

The biological effect of action of an ionizing radiation depends 1) on a kind of an ionizing radiation, 2) on exposure dose, 3) on time of action, the size of the irradiated surface and individual sensitivity of human organism. Radiations any kinds are dangerous to human organism. Blood and cells of hemopoietic organs are most sensitive to irradiation. Therefore the first indication of a radiation injury is the modification of composition of blood. Ability of a cell to division is broken at irradiation and growing organisms therefore are more strongly damaged.

The section of nuclear physics that studies the values describing action of ionizing radiation on substance and also methods and devices for its measurement is known as **dosimetry**. For estimation of radiation effect on a live organism, first of all for human organism is offered and used **the equivalent dose** of irradiation. Unit of the equivalent dose is 1 Sievert: 1 Sv=100 **ber (biological equivalent of roentgen** – stand-alone unit).

The maximum permissible biological dose for a person at professional irradiation is **5 ber** per year. For population it is **0.5 ber** per one year.

Minimal lethal dose is conditionally accepted ≈600 ber at irradiation of all body.

Devices for measurement of doses of ionizing radiation or the values connected with doses are called *dosimetric devices or dosimeters*. Dosimeter consists of 1) detector, 2) device of processing of information, 3) indicator of results.

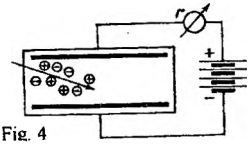


Fig. 4

In *detectors* energy of ionizing radiation transforms to electric signal. There are some detectors of radiation. The most widespread detectors are:1) **ionizing chamber; 2) counter of Geiger-Muller; 3) semiconductor and scintillometer detectors.**

In ionizing chamber absorbing substance is certain gas in the space between two electrodes (fig. 4). Particles getting into the chamber and photons of radiation cause occurrence of a current. The current is proportional to number of ions formed in the chamber per second and hence energy flow of transiting ionizing particles. Such chambers are used in the pocket dosimeters.

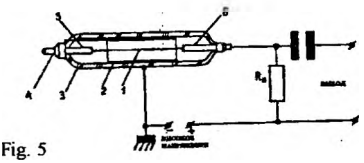


Fig. 5

The gas-discharge counters frequently termed as Geiger-Muller counters (fig. 5) differ from ionization chambers by the greater sensitivity and are capable to register individual pair of ions. By principle of the device such counter does not differ from ionizing chamber: it also is the condenser on which the potential difference is applied, however it is so great and that in the gas originates new process: gas amplification that is sharp magnification of initial number of ions (≈ in 10^7 times). The cylindrical Geiger-Muller counter consists of coaxially posed electrodes: 1 is the anode (the thin wire tensioned along an axis); 2 is the cathode as sprayed on the glass tube 3 metal; 4 is contact; 5 and 6 are insulators. Pressure

of gas inside the counter is about 100 mm Hg. To electrodes are applied the voltage at some hundreds volts. At hit into the counter of ionizing particle in gas are formed mobile electrons moving to the anode. As the wire is thin (diameter is about of 0.5 mm), then near to wire the field is strongly nonuniform, also intensity is great. Electrons near to wire are sped up so, that start to ionize gas. As result there is a discharge and on the circuit the current proceeds.

Device exposition

The dosimeter-radiometer АНРИ–01–02 "PINE" (further under the text – "device") is designed for individual use by the population for the purpose of the control of a radiation situation on terrain, in inhabited and workrooms. It is used for:

- Measurement of power exposition dose of γ - radiation in milliroentgens per hour (mR/h) from 0.01 to 9.999 or microsieverts per hour (mcSv/h) by multiplication of instrument readings on 10.
- Measurement of density of stream of β - radiation from the polluted surfaces in units: corpuscles / ($cm^2 \cdot min$) from 10 to 5000 or corpuscles / ($m^2 \cdot s$) from $1.66 \cdot 10^3$ to $8.33 \cdot 10^5$.
- Estimation of volume activity of radio nuclides in solutions (on an isotope ^{137}Cs) in the Curie per litre (Ku/l) from 10^{-7} to 10^{-6} or the Becquerel per litre (Bc/l) from $3.7 \cdot 10^3$ to $3.7 \cdot 10^4$. Necessary formulas are specified on the back cover of the device.

The basic relative accuracy of measurement for power exposition dose of γ - radiation on isotope ^{137}Cs is no more of ±30 %, stream density β - radiation from

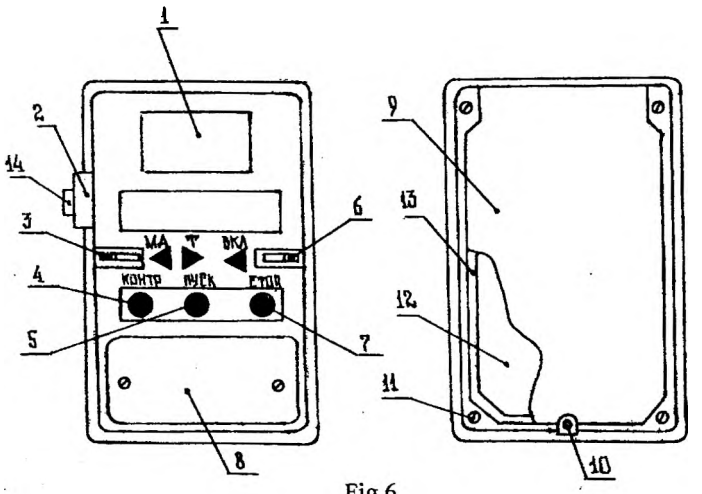


Fig.6

the firm plane source $^{90}Sr + ^{90}Y$ is no more ±45 %. The dosimeter is fulfilled in the form of the portable device, wearable on a thong or in clothes pouch (fig. 1).

In the upper on the obverse panel are placed controls and indications, at the bottom is compartment of device power with the cover. In the inferior part of the device is placed the board with the Geiger-Muller counters. The rotary back cover (which is the screening filter) fastens in the same place. On the case lateral surface there is the socket of connector 2 for connection of portable blocks of detection. By operation of the device from interior counters of ionization into this socket should be interposed plug 14. On the obverse panel of the device are had: the liquid crystal display 1; the switch of operating modes 3; the button of the control of working capacity of the device 4; the button "start" 5; the power switch 6; the button of "stop" 7; the cover of power 8.

On the back surface are had: back cover 9; holdfast of the back cover 10; place of seal integrity 11; the packing protective 12; framework 13.

By operation of the device the voltage transformer gives on the anodes of gas-discharge counters a voltage of ≈400 V. At hit to the working volumes of counters of ionizing particles, on the electrodes of counters there are impulses which are on the display 1. At position of the switch 3 "МД", in the device works the interior timer that through a preset time stops a pulse counting.

At "Т" position of the switch 3 the timer does not work. The count-down time is checked by a user on stop watch. On the display board the number of impulses for the given time is indexed. Switching off device is made by the switch 6 and should be accompanied by a short audible signal. If the device emits the steady signal it is necessary to put a new power.

At "МД" position of the switch 3 on the display is indexed four nulls and the point after the first sign. After pressure of the button 5 starts the counting out, that stops in 20±5 seconds, device gives the short signal.

If the switch 3 is in the "Т" position on the display at the switching of 6 are indexed four nulls without point. After pressure of the button 5 begins the count-down, every tenth impulse is accompanied by the audible signal. The count-down stops by pressure of the button 7. For execution of recurring measurements in any regimen, it is necessary to push the button 5.

Operation procedure

1. Preparation of the device for operation.

1. For testing of serviceability of electron scaling circuit and the device timer, lead the switch 3 in the position "МД", push the button "КОНТР" (to retain it in the pushed state till the end of control checkout), then transiently to push the button 5. The end of reading is accompanied by the short audible signal, and on the display 1.024 is indexed. After end of counting it is necessary to release the button "КОНТР". If at the control test the indexed number differs from specified above, the device is unworkable.

2. For testing of working capacity of the transformer of the voltage and counters, set the switch 3 in the position "МД" and push the button 5. After the finishing of measurement, on the display should be indexed the number closed to a natural

background of γ - radiation (≈20 mcR/h). If after testing on the display was fixed number 0.000 or number less than 0.005, the device is unworkable.

2. Device operation in the mode "поиск (search)".

1. Close the back cover 9 and set the switch 3 in the position "Т".
2. Switch the device the feed switch 6, push the button 5 and switch on the stop watch. In the minute push the button 7. Note instrument readings N_{Φ}. At natural background of γ - radiation the device should give 1-6 audible signals per minute. Value N_{Φ} should be subtracted from all subsequent instrument readings. Because of a stochastic (casual) pattern of activity of radioactive elements at low power of air (exposition) dose can be observed the appreciable dispersion in instrument readings, therefore it is necessary to take 3-5 indications and to compute mean value of background.
3. Bring the device to shoe and execute the item 2. Note instrument readings $N_{shoe+\Phi}$.
4. Fulfill items 2-3 with the open cover 9. The device in this case fixes impulses of β - and γ - radiation.
5. Close the cover 9, put it at height of 5-7 cm over the lead small house with radioactive preparation ($РП_1$) and open the cover РП. Switch on the device and transiently push the button 5 (start). Note instrument reading $N_{\gamma, h}$.
6. Fulfill item 5 with the open cover 9. Note instrument reading $N_{(\gamma+\beta), h}$.
7. Iterate items 5, 6, putting the device at height of h_2=40÷50 см. Note instrument readings $N_{\gamma, h2}$ and $N_{(\gamma+\beta), h2}$.
8. Locate between РП and the device the glass plate (board). Push the button 5 (start). Note instrument readings $N_{(\gamma+\beta), C}$ and $N_{(\gamma+\beta), k}$.

3. Device operation in the mode of measurement of power of air dose of γ - radiation.

1. Set the switch 3 in the position "МД" and fulfill item 5 of section II. Note instrument readings P_1. These readings are power of air dose of γ - radiation in mR/h.
2. Exchange $РП_1$ on $РП_2$ and push the button 5 (start). Note instrument reading P_2.

4. Device operation in the mode of measurement of stream density of β - radiation from low-purity surfaces.

1. Close the back cover of the device. Set the operating mode switch in the position "МД".
2. Bring the device by the plane of the back cover to an explored surface on the distance of 0.5-1 cm, switch on the device and transiently push the button 5 (start). Fulfill the measurement and note instrument readings N_{γ} without the comma account.
3. Fulfill measurement with the open back cover of the device to analogously to the point 2. Note instrument readings $N_{(\gamma+\beta)}$ without the comma account.
4. Close the back cover of the device and switch off the device.

5. Value of the stream density of β - radiation from the surface to compute by formula:

$$\Phi = K_S\left(N_{\gamma+\beta} - N_\gamma\right)\frac{particles}{sm^2 \cdot \min},$$

where K_S is coefficient of the account of the device $\left[\frac{particles}{cm^2 \cdot minute \cdot impulse}\right]$. $K_S = 0.5$.

5. Device operation in the mode of estimation of volume (specific) activity of radionuclides in a sample (probe).

Estimation of a volume (specific) radioactivity of a probe is the most complex operation by work with the device. Correctness of its estimation depends on many factors. By operation follows:

1. Take purely washed, dry basin from the complete set of the device and fill it to the mark "level" with pure potable water.
2. Open the back cover of the device and put it on the basin.
3. Set the switch 3 in the position "T"and switch on the device.
4. Fix the stop watch a time of the beginning of indication and simultaneously push the button 5 "start". In 2 or 5 minutes push the button 7 "stop". Note instrument reading N_Φ.
5. If the instrument reading is more than 1500 impulses, it is necessary to spend a decontamination of the basin and to repeat the measurement on the basin filled by water after decontamination. For decontamination it is necessary to flush carefully the basin in a solution of a washing-up liquid (a washing powder) and to wipe dry.
6. Fill the basin with investigated material on the mark "level". Solid materials are necessary make smaller and stacking in the basin the dense, equal stratum.
7. Put the device on the basin and to fulfill measurement analogously to items 2 – 4. Note instrument reading $N_{\Phi+П}$.
8. Switch of the device, take out it from the basin and close the back cover.
9. Estimation of volume activity of radionuclides is made by formula:

$$A = K_n\left(\frac{N_{\Phi+П}}{t_2} - \frac{N_\Phi}{t_1}\right)\frac{Cu}{l},$$

where t_1 is time of indication with the basin with water, minute; t_2 is time of indication with investigated material, minute; K_n is the device coefficient, it depends on characteristics of a measured material. Roughly coefficient K_n for the probes containing isotopes of ^{137}Cs is $8 \cdot 10^{-9}$ Cu·min / (l·Impulse). For calculation of volume activity in Bc/l it is necessary to accept $K_n = 3 \cdot 10^2$ Bc·min / (l·Impulse).

10. If as a result of indications and calculations to be gained quantity smaller than $2 \cdot 10^{-7}$ Cu/l, or that matches to a difference of instrument readings at two measurements of $N_{\Phi+П} - N_П < 250$ impulses, it is necessary to repeat the measurement having increased time of indication to $t_2 = 30$ minutes ± 10 seconds and to iterate evaluations. If as a result of recurring measurements and calculations was gained a quantity smaller than 10^{-7} Cu/l ($3.7 \cdot 10^3$ Bc/l), than to estimate the volume radioactivity is impossible, it is possible to consider only that $A < 10^{-7}$ Cu/l.

Control questions

1. Give the law of radioactive decay. Name the kinds of radioactivity.
2. Definition of units: Roentgen, Gray, Sievert, Becquerel, Curie.
3. Structure and principle of operation of the device.
4. Device operation in the mode of measurement of power of air dose.
5. Device operation in the mode of measurement of density of the stream of β - radiation from low-purity surfaces.
6. Device operation in the mode of estimation of volume activity of radionuclide in materials.

Test to the laboratory work №1

Methods of mathematical statistics for processing of medical and biological information

1. Which from below-enumerated random quantities belong to the *continuous* random variables?

1) number of letters on a page of the book; 2) arterial pressure of the patient;
3) temperature of the patient within a day; 4) pulse rate of a patient.

2. Mean is characteristic of:

1) variability of data in a sample; 2) most frequently meeting value of the random variable in the sample; 3) average arithmetic value.

3. Variance characterizes:

1) variability of data in a sample; 2) most frequently meeting value of a random variable in the sample; 3) average arithmetic value.

4. At increase of μ and σ the graph of a normal distribution law

1) is displaced to the right and becomes wider;
2) is displaced to the right and becomes more narrow;
3) is displaced to the left and becomes wider;
4) is displaced to the left and becomes more narrow.

5. What is f (x) in the Gauss law?

1) value of a random variable x; 2) standard deviation $\sqrt{D}$; 3) mean m;
4) density of probability dP/dx.

6. What does contain the table presenting a statistical interval series of distribution?

1) x_i, f(x) or μ; 2) m_i, f(x) or μ; 3) x_i, m_i or p_i; 4) x_i, h_i or μ.

7. What is the area under the histogram?

1) $S=\Sigma S_{rectangulars}$; 2) $S_{rectangular}$; 3) 0; 4) 1; 5) $dx \cdot f(x)$.

8. What is the random variable in our work?

1) body height of students in a group; 2) mass of student in a group;
3) pulse rate of students of a group; 4) distance from centre to centre of pupils of students of a group.

9. 50 measurements of a random variable are done. It is necessary to divide all values of random variables at plotting of a histogram on

1) 2 or 3 intervals; 2) 3 or 4 intervals; 3) 5 or 6 intervals; 4) 6 or 7 intervals.

10. How many random variables (%) hits into the interval $\mu \pm 3\sigma$?

1) 68; 2) 96; 3) 99.7; 4) 0.

Test to laboratory work №2

Testing of statistical hypotheses

1) Which of the next assumptions do not concern to statistical?

1) D(Y) = D(X); 2) M(X)=M(Y);

3) speed of gas molecules are distributed under the Maxwell law;

4) degree of disorder of a points is estimated by coefficient of correlation r.

2) Type I error is:

1) conclusion about existence of distinctions which actually are not present;

2) wrong decision of probability α;

3) correct decision of probability $1-\alpha$;

4) it is accepted H_1, when actually is true H_0.

3) Significance level α is:

1) probability of rejection of the null hypothesis, when H_0 is actually true;

2) correct decision of probability $1-\beta$;

3) type II error;

4) correct decision of probability $1-\alpha$.

4) Statistical test is:

1) number permitting according to sample to accept or reject a null hypothesis; 2) two-tailed critical region;

3) significance level α;

4) rule (formula) permitting according to sample to accept or reject H_0.

5) Let two samples are taken from population, which submits to known law of distribution, for example, from the normal law of distribution. What tests thus are used at testing of hypotheses?

1) Parametrical tests; 2) nonparametric tests; 3) Student's test;

4) F-test of Fisher; 5) Wilcoxon-Mann-Whitney U-test.

6) Observed value $k_{obs.}$ gets to critical region. What from hypotheses is necessary for accepting?

1) Depends from α; 2) H_0; 3) H_1; 4) H_0 or H_1.

7) Let p=0.03 and α=0.05. It means that:

1) distinctions are non significant, H_0 is accepted; 2) distinctions are significant, H_0 is accepted; 3) distinctions are non significant, H_1 is accepted; 4) distinctions are significant, H_1 is accepted.

8) It is necessary to compare means of two samples with equal variances from normal population. What test is necessary?

1) Student's test; 2) F-test of Fisher; 3) Wilcoxon-Mann-Whitney U-test.

9) It is necessary to compare means of two independent samples from population with unknown law of distribution. What test is necessary?

1) Student's test; 2) F-test of Fisher; 3) Wilcoxon-Mann-Whitney U-test.

Test for laboratory work №3:

Distribution fitting: testing of sample for normality

1. According to the central limit theorem, if any quantity is defined by the sum of big number of random factors and influence of each factor is insignificant, distribution of this quantity will be close to

1. uniform distribution; 2. Gauss distribution;
3. Maxwell distribution; 4. it can has any form.

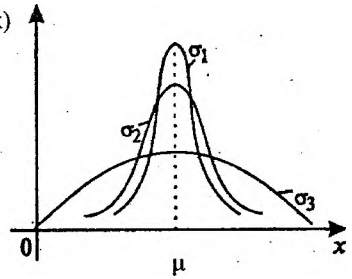
2. Probability density of normal distribution is formula

1. $f(x)=\frac{1}{\sigma\sqrt{2\pi}}\ell^{\frac{(x-\mu)}{2\sigma^2}}$ 2. $f(x)=\frac{1}{\sigma\sqrt{2\pi}}\ell^{\frac{(x-\mu)}{2\sigma}}$

3. $f(x)=\frac{1}{\sigma\sqrt{2\pi}}\ell^{\frac{(x-\mu)^2}{2\sigma}}$ 4. $f(x)=\frac{i}{\sigma\sqrt{2\pi}}\ell^{\frac{(x-\mu)^2}{2\sigma^2}}$

3. Specify a proper correlation for 1, 2 and 3 graphs of a normal Distribution (see fig.):

1. $\mu_3 > \mu_2 > \mu_1$, $\sigma_1 = \sigma_2 = \sigma_3$;
2. $\mu_3 = \mu_2 = \mu_1$, $\sigma_1 < \sigma_2 < \sigma_3$;
3. $\mu_3 = \mu_2 = \mu_1$, $\sigma_1 > \sigma_2 > \sigma_3$;
4. $\mu_3 = \mu_2 = \mu_1$, $\sigma_1 = \sigma_2 = \sigma_3$.



4. Qualitative (visual) checkout of testing of the data on normality are:

1. plotting of a histogram;
2. rule of three sigma;
3. probit-graps;
4. calculation of the coefficient of skewness, kurtosis and their standard errors;
5. using of goodness-of-fit tests.

5. For a graph of the normal distribution coefficient of skewness As and coefficient of kurtosis Ex are equal:

1. As =0, Ex=0; 2. $|As| > 0.5$, $|Ex| > 0.5$; 3. $As < 0$, $|Ex| \le 0.1$; 4. $As < 0$, $Ex > 0$.

6. If the significance level *p* at using of goodness-of-fit tests is $p < 0.05$, then:

1. null hypothesis about normality of sample is accepted and accordingly H_1 rejects;
2. H_0 about normality of sample is rejected; H_1 accepts;
3. H_0 about normality of sample is rejected; H_1 is rejected too.

7. W – test of Shapiro-Wilk is used if volume of sample is equal

1. $n > 30$; 2. $n > 50$; 3. $2 \le n \le 10$; 4. $3 < n < 50$.

8. Let in the W – test of Shapiro-Wilk $W_{obs.} < W_{cr.}$ Conclusion:

1. H_0 is rejected; 2. H_1 is rejected ; **3.** H_1 is accepted; 4. H_0 is accepted.

9. On any probit- graph is observed symmetrical deviation of points. Distribution is

1. far from normal; 2. close to normal;
3. normal; 4. additional analysis is necessary.

Test to laboratory work №4:

Testing of statistical hypotheses: chi square test

1. Point the basic properties of the χ^2 distribution:

1) It is not symmetric. 2) $\chi^2 > 0$, total area under the curve is equal to 1.

3) It is asymptotic to the y- axis on the right -hand-side.

4) The shape of the χ^2-distribution depends on degrees of freedom df.

5) If *df* increases, the χ^2-distribution becomes more symmetric.

2. What is the data type of the variable X: Pain degree (strong, moderated, mild pain, absence of a pain)?

1) categorical; 2) numerical; 3) ordinal; 4) discrete; 4) continuous.

3. Let we study unusual distribution of blood groups in patients after the certain surgical procedure. We know that expected distribution for population after this surgery is 44% group 0, 45% group A, 8% group B and 3% group AB. We know observed frequencies for each blood group too. What type of the chi square test we will use to analyze this data?

1) test of homogeneity; 2) test of independence; 3) test of goodness of fit.

4. What is the formula for test-statistic of the chi square?

1) $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$, 2) $\chi^2_{obs.} = \sum \frac{(|O_i - E_i| - 0.5)^2}{E_i}$,

3) $r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$, 4) $\chi^2 = \sum \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$.

5. How many degrees of freedom have the next contingency table?

	Heart rate increased	No heart rate increased	Total
Treated	36	14	50
Not treated	30	25	55
Total	66	39	105

1) 1; 2) 2; 3) 3; 4) 4.

6. Yates correction can be used for more exact calculations of χ^2 only 3

1) for df =1; 2) df = 2; 3) for tables 2x2; 4) for tables 2x3.

7. Formulate the null hypothesis for the test of independence:

1) H_0: the random variable follows the claimed distribution.

2) H_0: the row variable and column variable are dependent. 5

3) H_0: two or more samples have the same distribution

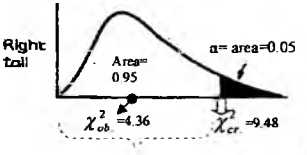
4) H_0: the row variable and column variable are independent.

8. Give the conclusion using the picture.

We reject next hypothesis:

1) H_0; 2) H_1; 3) we accept H_0;

4) we accept H_1.



9. Give the conclusion, if $\chi^2_{obs.} > \chi^2_{cr.}$:

1) we accept H_0; 2) we accept H_1;

3) we reject H_0; 4) we reject H_1.

Test to laboratory work №5:

Use of elements of the correlation analysis and the method of the least squares at processing of medical and biological information

1. Specify examples of the correlation relationship between quantities:
1) dose of a medicine and its concentration in blood;
2) diameter of a vessel and the volume of blood flowing through it;
3) time spent for preparation to a laboratory work and the obtained mark;
4) velocity of a moving erythrocyte and a shift of frequency in the Doppler effect.

2. Coefficients *a* and *b* in the equation of regress Y on X: $M(Y_x)=ax + b$= (Slope)·(x)+(y-intercept) are possible to find
1) by method of least squares;
2) on the stipulation that sum of squares of deviations of ordinates of experimental points from ordinates of points of a smoothing straight line would be **minimal;**
3) on the stipulation that sum of squares of deviations of ordinates of experimental points from ordinates of points of a smoothing straight line would be **maximal.**

3. Nature of correlation relationship and character of disposition of the points on a scatter diagram is estimated by the
1) force of relationship; 2) coefficient of correlation r;
3) form of the equation of relationship.

4. Coefficient of regress ρ_{yx} (or coefficient "a" - slope) in the equation of regress is estimated by
1) force of relationship; 2) coefficient of correlation r;
3) form of the equation of relationship.

5. Degree of disorder of the points of correlation relationship is estimated by the
1) force of relationship; 2) coefficient of correlation r;
3) form of the equation of relationship.

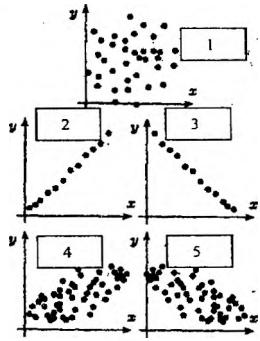


Fig. 1

6. Which of scatter diagrams (fig. 1) are showing absence of linear correlation relationship?
1) 1; 2) 2; 3) 3; 4) 4; 5) 5; 6) 6.

7. Coefficient of correlation *r* estimates
1) force of relationship; 2) degree of disorder of the points ;
3) form of the equation of relationship.

8. For which diagram (see fig. 1) r <0 and functional simultaneously?
1) 1; 2) 2; 3) 3; 4) 4; 5) 5; 6) 6.

9. Check of significance of r is made as
1) r is random variable; 2) r can be small quantity;
3)X and Y are random variables; 4) r can be the big quantity.

10. For which diagram (fig. 1) r > 0 and r ≈ 0.5 simultaneously?
1) 1; 2) 2; 3) 3; 4) 4; 5) 5; 6) 6.

Test to laboratory work №6:
Study of the Spearmen's coefficient of correlation

1. Point the correct assumptions of application of the Spearmen's Rho:
1) at least one of variables x or y is measured in rank (ordinal) scale;
2) distribution at least one of variables does not submit to the normal law;
3) connection between x and y is non-linear (but monotonous);
4) the size of sample is small;
5) both variables are measured in rank (ordinal) scale.

2. Select the formula for calculation of Spearmen's ρ:

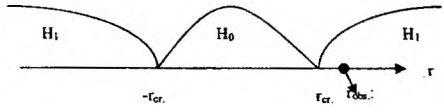
1) $t_{obs.} = \frac{r_{obs.}\sqrt{n-2}}{\sqrt{1-r_{obs.}^2}}$; 2) $r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$;

3) $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$, 4) $r_x = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{s_x s_y}$.

3. Properties and characteristics of Spearmen's ρ are:
1) $|r| \le 1$; 2) if ranks are equal for all x_i and y_i, then $r_s = 1$; 3) $n \le 50$;
4) r_x^2 represent a share of variation of y that it is possible to explain by changing of x;
5) $r_{obs.}$ is computed by formula and r_{crit} is we find under the tables.

4. Let $|r_{obs.}| > r_{cr.}$. We accept:
1) H_1; 2) H_0; 3) H_0 is rejected; 4) H_1 is rejected.

5. Give the conclusion under the picture. We accept:
1) H_1; 2) H_0;
3) H_0 is rejected;
4) H_1 is rejected.



6. If the sample size is more 50, as test-statistic we use the formula:
Look the answers of the question 2.

7. Let the significance level is α=0.05, sample size is 7, then $r_{cr.}$ is
1) 0.929; 2) 0.786; 3) 0.738; 4) 0.700.

8. Let the significance level is α=0.05. p=0.23 (calculated by the package Statistica). We accept:
1) H_1; 2) H_0; 3) H_0 is rejected; 4) H_1 is rejected.

9. The advantages of the Spearmen's rho are:
1) Big sensitivity; 2) simplicity of calculation;
3) fast way of rejection of the null hypothesis;
4) use for the normal distribution.

Test to laboratory work №7

Theory of probability in medicine: conditional probability; composite probability formula; Bayes's theorem

1) Two coins are tossed simultaneously. If one of them turned head, what is the probability that the other one turned head?
1) 0.01; 2)0.05; 3) 0.025; 4) 0.50.

2) Let A and B are two events such that P(B)=1/4, P(A/B)=1/3, what will be P(B·A)?
1) 1/6; 2) 1/12; 3) 3/2; 4) 2/3.

3) Three coins are tossed. What is the total number of cases (find the sample space)?
1) 3; 2) 6; 3) 8; 4) 9.

4) Three coins are tossed. What is the probability of getting of all heads?
1) 1/3; 2) 1/6; 3) 1/8; 4) 1/9.

5) Give the composite probability formula:

1) $P(A) = \sum_{i=1}^{n} P(H_i) \cdot P(A/H_i)$; 2) $\sum P(H_{ii}) \cdot P(A/H_i)$;

3) $P(H_i/A) = \frac{P(H_i)}{\sum P(H_i) \cdot P(A/H_i)} \cdot P(A/H_i)$; 4) $\sum P(H_{ii}) = 1$.

6) Give the formula of Bayes` theorem:
See the previous answers.

7) Three urns are given each containing red and white balls. Urn I contains 6 red and 4 white balls, urn II contains 2 red and 6 white balls and urn III contains 1 red and 2 white balls. An urn is selected at random and a ball is drawn. What is the probability that is drawn a red ball?
1) 0.24; 2) 0.39; 3) 0.48; 4) 0.51.

8) If the ball is red what is the probability that it is from the first urn?
See the previous answers.

9) What is the sum of all a priory probabilities?
1) 0; 2) 1/3; 3) ½; 4)1.

Test to laboratory work №8

DETERMINATION OF THE MOMENT OF INERTIA OF BODIES AND CHECK OF THE BASIC LAW OF DYNAMICS OF ROTATIONAL MOTION

1) Moment of inertia I of a material point of mass m is equal
1. m r; 2. Σ mr; 3. mr^2; 4. Σ mr^2; 5. Fr

2) Unit of measurement of the moment of inertia in SI is:
1. kg·m; 2. kg·m^2; 3. N·m; 4. N·m^2; 5. kg/m

3) The module of moment of force (torque) is equal (fig. 1):
1. F d; 2. F r; 3. F d^2; 4. F d sin α; 5. F·r sin α

4) Angular (rotary) acceleration β is
1. $\frac{d\varphi}{dt}$; 2. $\frac{d\beta}{dt}$; 3. $\frac{dr}{dt}$; 4. $\frac{dm}{dt}$; 5. $\frac{d\omega}{dt}$

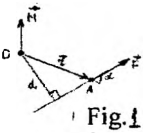


Fig.1

5) Define the kind of dependence between rotary acceleration and moment of inertia of a body at constant value of the moment of force:
1. linear; 2. square-law; 3. exponential;
4. directly proportional; 5. inversely proportional

6) At Oberbek pendulum mass of the load on the extremity of cord have increased twice, not changing position of loads on all rods. Moment of inertia
1. will increase in 2 times; 2. will increase in 4 times; 3. practically will not change; 4. will decrease in 2 times; 5. will decrease in 4 times

7) If to translocate loads of the pendulum to the ends of rods (M=const), moment of inertia of the pendulum
1. will increase; 2. will decrease; 3. will not change; 4. becomes equal to null

8) For determination of moment of inertia in our work it is necessary to measure
1. m,t,r,h; 2. M, β; 3. m,t,r,g,h ; 4. α,β,m,r,t,h

9) The ice skater fulfills rotation and holds down his hand to his body. How will change moment of inertia I of the body of the ice skater and the rotary acceleration of his gyration β?
1. **I** will increase, **β** will decrease; 2. **β** will increase, **I** will decrease;
3. **I** will increase, **β** will increase; 4. **I** will decrease, **β** will decrease

Test to laboratory work №9
Study of oscillatory motion with help of kimograph

1. Harmonious oscillations are:
1) Oscillations which are made under the law of cos or sine;
2) Addition of two oscillations of identical directions with poorly distinguished frequencies;
3) Periodically repeating oscillations;
4) Addition of two oscillations of opposite directions.

2. Period of oscillations is:
1) Number of oscillations made per unit of time;
2) Time during which one oscillation is made;
3) Time during which the amplitude decreases in e times;
4) Number of oscillations made in time T.

3. The differential equation of damped harmonious oscillations is:

1) $\frac{d^2x}{dt^2}+\omega_0^2x=0$;
2) $\frac{d^2x}{dt^2}+2\beta\frac{dx}{dt}+\omega_0^2x=0$;
3) $2\beta\frac{dx}{dt}+\omega_0^2x=0$;
4) $\frac{dx}{dt}+2\beta x=0$.

4. Find the coefficient of attenuation β (s^{-1}), if: A_t =30 mm; A_{t+10T}=15 mm; T=1.2 s.
1) 0.325; 2) 0.058; 3) 0.581; 4) 0.012.

5. Find the initial phase of φ_0 of the damped harmonious oscillations, if it is given: l=10mm; l_0=5 mm.
1) 0.3 π; 2)0.56 π; 3)π; 4)1.8 π.

6. Expression of amplitude of damped harmonious oscillations is:

1) $A_0\cdot e^{-\beta t}$;
2) $x=e^{-\beta t}$;
3) $2A_0\cos\frac{\omega_1-\omega_2}{2}t$;
4) $\ln e^{\beta t}$.

7. The equation of displacement of damped harmonious oscillations is:

1) $x=2A_0\cos\frac{\omega_1-\omega_2}{2}\cdot\cos\frac{\omega_1+\omega_2}{2}t$;
2) $x=2A_0\cos(\omega t+\phi_0)$;
3) $x=A_0\cdot e^{-\beta t}\cdot\cos(\omega t+\phi_0)$;
4) $x=2A_0\cdot e^{-\beta t}$.

8. Beats are:
1) Oscillations which are made under the law of cos or sine;
2) Addition of two oscillations of identical directions with poorly distinguished frequencies;
3) Periodically repeating oscillations;
4) Addition of two oscillations of opposite directions.

9. The equation of displacement of beats is:

1) $x=2A_0\cos\frac{\omega_1-\omega_2}{2}\cdot\cos\frac{\omega_1+\omega_2}{2}t$;
2) $x=2A_0\cos(\omega t+\phi_0)$;
3) $x=A_0\cdot e^{-\beta t}\cdot\cos(\omega t+\phi_0)$;
4) $x=2A_0\cdot e^{-\beta t}$.

10. Write down the equation of the beats, if it is given: ω_1=1.82 π; ω_2=1.8 π; A_0=10.
1) $30\cos 0.02t\cdot\cos 1.7\pi t$;
2) $30\cos 0.01t\cdot\cos 1.8\pi t$;
3) $20\cos 0.01t\cdot\cos 1.8\pi t$;
4) $20\cos 0.03t\cdot\cos 2.1t$.

Test to laboratory work №10

Study of mechanical properties of a bone tissue

1. What are the basic kinds of deformations?

1) stretching; **2)** compression; **3)** bend; **4)** shift;
5) torsion; **6)** sliding; **7)** tension.

2. What are the percent elongation and absolute elongation?

1) $\varepsilon = \frac{\Delta\ell}{\ell}$, $\Delta = l_1 - l_2$; **2)** $\varepsilon = \frac{\Delta\ell}{\ell}$, $\Delta = l_2 - l_1$;

3) $\varepsilon = \frac{\Delta\ell}{\ell_1}$, $\Delta = l_2 - l_1$; **4)** $\varepsilon = \frac{\Delta\ell}{\ell_2}$, $\Delta = l_2 - l_1$.

3. Give the formula of Hooke's law:

1) $\sigma = \frac{F}{S}$; **3)** $\sigma = \frac{\varepsilon}{E}$;

2) $\sigma = E \cdot \varepsilon$; **4)** $\varepsilon = \frac{\Delta l}{l}$.

4. Formula of Young's modulus for the samples in form of the tube is:

1) $E = \frac{P \cdot l^3}{12\pi f (R^4 - r^4)}$; **3)** $E = \frac{\sigma}{S}$;

2) $E = \frac{P \cdot l^3 (R^4 - r^4)}{12\pi f}$; **4)** $E = \frac{P \cdot l^3}{12\pi f (R^2 - r^2)}$.

5. Unit of measurement of the module of elasticity (Young's modulus) is:

1) N/m^2; **2)** N; **3)** Pa; **4)** Pa/m^2.

6. Give the unit of the bending deflection:

1) m; **2)** Pa; **3)** N; **4)** kg.

7. We study in the our work dependence of:

1) sag on load; 2) sag on displacement of the middle of sample;
2) Young's modulus on load;
3) Young's modulus on displacement of the middle of the sample.

8. Young's modulus is numerically equal to the:

1) force working per unit of the area of cross-section;
2) force working per unit of length;
3) pressure (mechanical) at which $\varepsilon = 1$;
4) pressure at which the length of the sample increases in 2 times.

9. Sample is considered more elastic, if it has (for the same loads and sizes):

1) larger Young's modulus and larger sag (f);
2) smaller Young's modulus and larger sag (f);
3) smaller Young's modulus and smaller sag (f);
4) larger Young's modulus and smaller sag (f).

Test to laboratory work №11:

Spectral characteristic test of ear on threshold of audibility

1. **Physical characteristics of sound are:**
   - **1)** loudness;
   - **2)** intensity;
   - **3)** frequency;
   - **4)** wavelength;
   - **5)** timbre;
   - **6)** height of tone.

2. **Physiological characteristics of acoustical sensation are:**
   - **1)** loudness;
   - **2)** intensity;
   - **3)** frequency;
   - **4)** wavelength;
   - **5)** timbre;
   - **6)** height of tone.

3. **Select the unit of the level of loudness:**
   1) Wt/m^2; **2)** Hz; **3)** Phon (Ph); **4)** Bell (B); **5)** dB;

4. **If intensity of a sound increased in 100 times, loudness of sound will increase:**
   - **1)** in 2 times;
   - **2)** on 20 phon;
   - **3)** in 10 times;
   - **4)** on 100 phons;
   - **5)** in 100 times.

5. **Law of Weber–Fehner is:**
   1) $E = \kappa \lg \frac{I}{I_0}$; **2)** $E = \lg \frac{I}{I_0}$; **3)** $E = -k \lg \frac{I}{I_0}$; **4)** E=kL.

6. **Decibel (db) is unit of:**
   - **1)** level of loudness;
   - **2)** intensity of sound;
   - **3)** sound pressure;
   - **4)** level of intensity.

7. **Sound frequencies perceived by ear are in the range of:**
   - **1)** 16 Hz–20000 Hz;
   - **2)** 20000 Hz–10^{10} Hz;
   - **3)** 10^{-3} Hz–20 Hz;
   - **4)** 16 Hz–10^{10} Hz.

8. **In the given work we study dependence of:**
   1) Level of intensity on the threshold of audibility on the log of frequency;
   2) Level of loudness on the threshold of audibility on the log of frequency;
   3) Level of intensity on the voltage.

9. **Maximal sensitivity of a human ear is placed in the range of frequencies:**
   - **1)** 1000-3000 Hz;
   - **2)** 0-1000 Hz;
   - **3)** 500-1000 Hz;
   - **4)** 5000 -10000 Hz.

Test to laboratory work №12

Study of action of ultrasonic oscillations on substance and determination of wavelength and speed of propagation of ultrasound (US)

2. Ultrasound is:
1) electromagnetic wave of frequency from 20 kHz;
2) mechanical waves of frequency <16 Hz;
3) electromagnetic waves of frequency < 16 Hz;
4) mechanical waves of frequency from 20 kHz up to 10^{10} Hz.

3. In medicine ultrasound is generated with help of the
1) direct piezoeffect; 3) phenomenon of magnetostriction;
2) converse piezoeffect; 4) photo-electric effect.

4. Amplitude of oscillations of piezoelement is maximal, if own frequency of the plate will be:
1) equal to frequency of the generator;
2) less than frequency of the generator;
3) more than frequency of the generator.

5. What are the basic actions of ultrasound on substance?
1) thermal; 2) biological; 3) mechanical;
4) electromagnetic; 5) physical and chemical;

5. Effect of magnetostriction is based on changing of the size of the ferromagnetic rod under the action of:
1) alternating magnetic field; 3) alternating current;
2) radiation; 4) compression of the ferromagnetic core.

6. What are the basic features of propagation of ultrasound in a medium?
1) big length of a wave; 2) small length of a wave;
3) can be focused easily; 4) propagated on significant distances;
5)speed of propagation and absorption strongly depends on properties of the medium;

7. Surface of a body at ultrasonic research is covered by vaseline for:
1) decreasing of reflection of US; 4) increasing of heat conductivity;
2) increasing of reflection of US; 5) decreasing of absorption of US.
3) increasing of electroconductivity;

8. Particles of starch at action of ultrasound are concentrated
1) in the antinodes of standing wave; 3) in the nodes of standing wave;
2) on the distance of $\lambda/2$; 4) on the distance of λ.

9. We can determine length of the wave of ultrasound in the given work under the formula:

1) $\lambda = \frac{2h}{n}$; 2) $\upsilon = \lambda\nu$; 3) $2d\sin\alpha = n\lambda$; 4) $\lambda = \frac{n}{2h}$.

Test to the laboratory work №13

Biophysical bases of measurement of arterial pressure

1. **Value of BP in any point of vascular system depends on**
   1) change of initial pressure p_0 in the heart, 2) change of volumetric flow rate Q, 3) change of resistance of vessels Z.
2. **Most exact method of BP measurement is method**
   1) of Riva-Rocci, 2) oscillometric, 3) direct method,
   4) Korotkov's method (for indirect measurements).
3. **Basic devices at measurements of BP by Riva-Rocci method are**
   1) phonendoscope, 2) sphygmomanometer,
   3) cuff, 4) mercury manometer.
4. **Basic physical idea of the Korotkov's method is**
   1) pressure of air in the cuff is less of pressure in the hand tissues,
   2) pressure of air in the cuff equals to pressure in the hand tissues,
   3) pressure of air in the cuff is more of pressure in the hand tissues.
5. **Korotkov's sounds are stipulated by**
   1) motion of heart walls, 2) vibration of arteria and forming of shock wave, 3) sounds caused by a turbulence of blood flow.
6. **Fast deflation of air from the cuff tends to**
   1) decrease of diastolic and to increase of systolic pressure,
   2) decrease of systolic and to increase of diastolic pressure,
   3) increase of systolic and to increase diastolic pressure.
7. **The cuff should cover**
   1) > 20 % of round of the shoulder,
   2) > 40 % of round of the shoulder,
   3) > 60 % of round of the shoulder,
   4) > 80 % of round of the shoulder.
8. **Pulse pressure is**
   1) difference between the SAP and DAP,
   2) pressure increasing when the vessel is squeezed,
   3) difference between the lateral SAP and DAP.
9. **Accuracy rating A/A of the given BP device means that**
   1) > 60 % of all measurements of its indications differ from indications of the reference instrument (known good device) less than on 5 mmHg,
   2) > 85 % of all measurements of its indications differ from indications of the reference instrument less than on 10 mmHg
   3) > 95 % of all measurements of its indications differ from indications of the reference instrument less than on 15 mmHg.

Test to the laboratory work №14:

Determination of the surface tension of liquids by the method of measurement of maximal pressure in the air bubble

1. **Reason of the surface tension is distinction of:**
   1) temperatures of mediums; 2) forces of interaction;
   3) speed of movement of molecules of mediums;
   4) densities of mediums.

2. **Give the SI units for the surface tension coefficient:**
   1) Pa·s; 2) N· m; 3) J/m^2; 4) N/m.

3. **Additional pressure in the air bubble caused by surface tension is equalized by:**
   1) atmospheric pressure; 2) dynamic pressure;
   3) static pressure; 4) hydrostatic pressure.

4. **Give the formulas for definition of the surface tension coefficient:**
   1) $\alpha = \frac{F}{l}$; 2) $\alpha = \frac{A}{l}$; 3) $\alpha = \frac{F}{S}$; 4) $\alpha = F \cdot l$; 5) $\alpha = \frac{A}{S}$.

5. **The working formula for determination of a surface tension coefficient by the Rebinder's method is:**
   1) $\alpha = \alpha_0 \frac{\Delta h}{\Delta h_0}$; 2) $\alpha = \alpha_0 \frac{\Delta h_0}{\Delta h}$; 3) $\alpha = F \cdot l$; 4) $\alpha = \rho g \Delta h$.

6. **Pressure in the air bubble arising on the end of capillary is equal:**
   1) p+Δp; 2) p-Δp; 3) p_0; 4) $p_0 + \Delta p$; 5) $p_0 - \Delta p$; 6) Δp.

7. **Substances are called the surface-active, if they are:**
   1) increase the α ; 4) increase the viscosity of a solution;
   2) decrease the α ; 5) change the temperature of a solution.
   3) do not influence on the α;

8. **Formula of pressure caused by surface tension (Laplace's formula) is:**
   1) $p = \frac{2\alpha}{R}$; 2) $p = 2\alpha R$; 3) $p = fg\Delta h$; 4) $p = \frac{2R\Delta h}{\alpha}$.

9. **Force of surface tension is directed:**
   1) on the tangent to the surface of a liquid;
   2) perpendicularly to the surface of a liquid;
   3) perpendicularly to the line of force action;
   4) on the tangent to the line of possible break.

Test to laboratory work №15

Determination of viscosity of a liquid with the help of viscosimeter

1. **Liquids are called newtonias, if:**
   1) its viscosity depends on gradient of speed;
   2) its viscosity depends on speed of flow;
   3) does not submit to the equation of Newton;
   4) its viscosity does not depend on gradient of speed.
2. **Unit of viscosity in SI is:**
   1) Pa·s **4)** $m^2/кg$
   2) $кg/м^3$ **5)** dimensionless size.
   3) N/m^2
3. **Working formula for determination of viscosity by Ostvald's method is:**

   1) $\eta = \eta_0 \frac{\rho t}{\rho_0 t_0}$; **2)** $\eta = \eta_0 \frac{\rho_0 t_0}{\rho t}$; **3)** $\eta = \frac{\eta_0 - \eta}{\eta}$; **4)** $\eta = \eta_0 \frac{\rho_0 t}{\rho t_0}$.

4. **Formula of Poiseuille is:**

   1) $V = \frac{\pi r^4 \Delta p}{8\eta l}$; **2)** $V = \frac{\pi r^4 \Delta pt}{8\eta l}$; **3)** $V = \frac{8\eta l}{\pi r^4 \Delta pt}$; **4)** $V = \frac{\pi r^4 \Delta pt}{8\eta s}$.

5. **Viscosity is:**
   1) ability of liquid to resist its compression; **3)** fluidity of a liquid;
   2) measure of resistance of a liquid to flow;
   4) force of internal friction between layers of liquid per unit of area and gradient of speed.

6. **At growth of temperature viscosity of a newtonian liquid will be:**
   1) increase; **2)** decrease; **3)** the constant;
   4) for some liquids decrease, for others increase.

7. **In the given work we study dependence:**
   1) viscosity of liquid on temperature;
   2) viscosity of liquid on concentration;
   3) coefficient of surface tension on temperature;
   4) viscosity on density of the liquid.
8. **Newton's formula for viscosity of liquid is:**

   1) $F = \eta \frac{dv}{dx} S$; **3)** $F = \eta \cdot \frac{dv}{dx} \cdot T$;

   2) $F = \eta v\ S$; **4)** $\eta = F \cdot S \cdot \frac{dr}{dv}$.

9. **Viscosity of blood in norm is (cPuas):**
   1) 5-10; **3)** 4-5;
   2) 1.7-22; **4)** 10-15.

Test to laboratory work №16

Determination of EMF by the method of compensation

1. EMF of a source is:

1) the source of energy of not electric nature;
2) the potential difference on the poles of a source at disconnected of the external circuit;
3) work of not electrostatic forces on moving of the unit of charge Along closed circuit.

2. EMF of the researched source is compensated by:

1) EMF the known source;
2) the voltage on the part of the slide-wire;
3) force of the current proceeding on slide-wire.

3. For determination of unknown EMF it is necessary to measure:

1) E_0, l_0, l; 2) E, l, l_0; 3) E, E_x, E_0; 4) E_x, l, l_0.

4. Working formula for calculation of the unknown EMF is:

1) $E_x = \frac{l}{E_0 l_0}$; 2) $E_x = E_0 \frac{l_0}{l}$; 3) $E_x = E_0 \frac{l}{l_0}$; 4) $E_x = E_0 \frac{l_0}{l^2}$.

5. The value of voltage is:

1) work done by electrostatic and external forces at moving of the unit of positive charge;
2) work done by electrostatic forces on the site of a circuit;
3) work done by external forces on the site of a circuit.

6. Work of the electrostatic forces for the closed circuit equals:

1) 0; 2) 1; 3) $\varphi \cdot q$; 4) A/q.

7. The Ohm's law for the part of circuit (absence of EMF) is:

1) $I = \frac{\varphi_1 - \varphi_2}{R}$; 2) $I = \frac{(\varphi_1 - \varphi_2) + E}{R}$; 3) $I = \frac{E}{R + r}$.

8. The Ohm's law for the closed circuit (with EMF) is:

1) $I = \frac{\varphi_1 - \varphi_2}{R}$; 2) $I = \frac{(\varphi_1 - \varphi_2) + E}{R}$; 3) $I = \frac{E}{R + r}$.

9. Point the correct equalities at determination of EMF:

1) $U_{exter} = E - Ir$; 2) $E = IR = U_{exter}$; 3) $E = IR$, if $I=0$; 4) $E_x = I R_x$.

Test to laboratory work №17

Study of the electric dipole field

1. Intensity (electric field strength) of the electric field is:

1) $E = \frac{A}{q}$; **2)** $E = \frac{F}{q}$; **3)** $E = Fq$; **4)** $E = Aq$.

2. Equpotential lines are lines:

1) that coincide to lines of intensity of the electric field;
2) of identical potential; **3)** of identical intensity;
4) connecting two charges.

3. Dipole moment $\bar{P}$ of a charging dipole is directed:

1) from plus to minus; **3)** at right angle to the arm of the dipole;
2) from minus to plus; **4)** at sharp angle to the arm of the dipole

4. Projection of the dipole moment of the electric vector of heart is proportional:

1) to the arm of a dipole; **2)** to the voltage between 2 points;
3) to cos α; **4)** to the distance r up to the point of measurement.

5. Potential in any point A of the current dipole field equals:

1) $\varphi = \frac{D}{4\pi r^2}\cos\alpha_A$; **2)** $\varphi = \frac{D\rho}{4\pi r^2}\cos\alpha_A$; **3)** $\varphi = \frac{D\rho}{4\pi r}\cos\alpha_A$; **4)** $\varphi = \frac{D\rho}{4r^2}$.

6. Lead is:

1) 2 points on the surface of a human body;
2) Δφ between 2 points on a human body;
3) current between 2 points on a human body.

7. Electrocardiogram is dependence:

1) I=f (t); **2)** R=f (U); **3)** U=f (t); **4)** U=f (I).

8. According to Einthowen's theory model of heart is:

1) unipole; **2)** charging dipole; **3)** current dipole; **4)** multipole.

9. Electrodes superimposed on the human body surface at ECG are designed for taking of:

1) charges generated on the human body surface;
2) current between 2 points on the body surface;
3) potential differences between 2 points on the body surface.

10. Second lead of ECG is:

1) LA-LF; **2)** RA-LF; **3)** RA-RF; **4)** RA-LA.

Test to laboratory work №18

Determination of resistance of a human body to the direct and alternating current

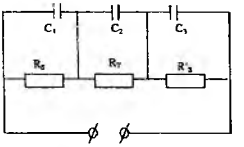
1. Tissue of a human organism has the next resistance:

1) inductive; 3) ohmic;
2) capacitive; 4) all mentioned above resistances.

2. At flowing of the direct current through a live tissue the current:

1) increases; 2) decreases; 3) remains the constant;
4) in the beginning decreases and then remains the constant.

3. Point elements of the equivalent circuit of a live tissue (see fig.) at passage of the alternating current:



1) R_S, C_2, R_S; 2) C_1, C_2, C_3;
3) C_1, C_2, R_T, C_3; 4) R_S, R_T, R_S.

4. Point elements of the equivalent circuit of a live tissue (see fig.) at passage of the direct current:

1) R_S, C_2, R_S; 2) C_1, C_2, C_3;
3) C_1, C_2, R_T, C_3; 4) R_S, R_T, R_S.

5. Impedance of a human tissue can be determined by the formula:

1) $Z = \sqrt{R^2_T + \left(\frac{1}{\omega C}\right)^2}$; 2) $Z = \sqrt{R^2_T + (\omega L)^2}$; 3) $Z = \sqrt{R_T + \left(\frac{1}{\omega C}\right)}$.

6. For determination of skin resistance to the direct current it is necessary to measure:

1) U, I, R_s; 2) I, R_s,C_1; 3) R_s, R_T,U; 4) U, I.

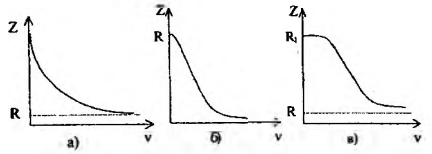
7. What values are necessary to measure for determination the live tissue resistance on alternating current?

1) R_T, U, C; 2) U, I, R_T; 3) R_T, C_1, C_2, C_3; 4) U, I.

8. At increase of frequency total resistance of tissue tends to:

1) R_T; 2) R_C; 3) $R_T + R_C$; 4) $\frac{1}{\omega C}$.

9. What is the correct dependence of Z=f(v) for a human tissue?



1) a; 2) b; 3) c.

Test for laboratory work №19

Determination of parameters of electric impulses

1. Impulse signal key parameters are:

1) amplitude; **2)** period of recurrence T; **3)** frequency of recurrence $\nu = \frac{1}{T}$;

4) pulse duration τ_u; **5)** relative pulse duration $Q = \frac{T}{t_u}$;

6) rate of pulse rise of front S_ϕ; **7)** duty factor $K = 1/Q = \nu\tau_u$.

2. Key parameters of a pulsing (impulse) current (or voltage) are:
See the previous answers.

3. Threshold current is:
1) minimal current at which occurs reducing of muscles (traction);
2) minimal perceptible current;
3) minimal current at which occurs the long-lived traction.

4. By the law Du Bois-Reymond traction of a muscle is directly proportional:
1) to the period of an impulse; **2)** to relative pulse duration of an impulse;
3) to pulse amplitude; **4)** speed of change of the current.

5. As follows from the equation of Weiss – Lapik, the rheobase is:
1) the minimal threshold current necessary for irritation at the long-lived action of a current;
2) a charge, which is necessary for passing to cause excitation at very short time of action;
3) the minimal time necessary for excitation of muscles.

6. Chronaxia characterizes:
1) level of excitability of a tissue;
2) time necessary for exitation at the current equal two rheobases;
3) the minimal force of the threshold current.

7. Electrical stimulation is:
1) application of impulse currents with the purpose of excitation or amplification of activity of particular organs, muscles and nerves;
2) medical action of pulsing and alternating current on biologically active points;
3) passing of high-frequency currents through tissues at physiotherapeutic procedures.

8. Square-wave pulses are used in
1) diadynamic therapy; **2)** transcranial electroanalgesia;
3) defibrillation; **4)** electrodiagnostic.

9. At the electrical stimulation of heart usually are used impulses:
1) of the triangular shape of frequency 100 Hz;
2) of the exponential shape of ν up to 160 Hz, I = 1-8 mA;
3) of the rectangular shape of ν = 1-1.2 Hz, I = 5-15 mA.

Test to laboratory work №20
Study of the galvanizing apparatus

1. What property of the semi-conductor diode is used for rectification?
1) bilateral conductivity; 2) unilateral conductivity;
3) smoothing of the alternating current pulsing.

2. What elements are necessary for reception of the full-wave rectification?
1) transformer; 2) choke; 3) 4 diodes;
4) 2 electrdes; 5) condenser; 6) oscillograph.

3. Smoothing filter consists of the next parts:
1) diode; 2) choke; 3) condenser; 4) transformer.

4. What is medical galvanizing?
1) method of medical influence by the alternating electric current;
2) method of medical influence by direct current up to 50 mA and by the voltage up to 80V;
3) method of medical influence by alternating magnetic field of 500 kHz.

5. Device of galvanizing is intended for generation:
1) of the direct current and realization of electrophoresis;
2) of the alternating current and treatment by AC;
3) both direct and alternating current.

6. The appreciable (min) current is equal (Amperes):
1) $(0.1 - 10)\cdot10^{-6}$; 2) $(0.1 - 10)\cdot10^{-5}$; 3) $(0.1 - 10)\cdot10^{-4}$; 4) $(0.1 - 10)\cdot10^{-3}$.

7. Ions of zinc at electrophoresis are entered at electrophoresis:
1) from negative electrode; 2) from positive electrode;
3) from the electrode of the same polarity.

8. Medicinal electrophoresis is a method of introduction into a human organism of medicinal substances with help of:
1) the direct current; 2) the alternating current; 3) the magnetic field.

9. What for at galvanizing is placed between the tissue and the electrode moistened cloth pad?
1) for effective work of the galvanizing device;
2) for achievement of optimum value of the direct current;
3) for exception of irritation and burns.

Test to laboratory work №21
Graduation of thermistor

1. What properties of the thermistor are used in the medical and biological practice?
1) sensitivity to change of pressure; 2) small size;
3) sensitivity to the changing of temperature;
4) sensitivity to change of capacity.

2. According to «the band (zones) theory», distinction between conductors, semiconductors and dielectrics are defined by the width:
1) of the valence band; 2) of the band gap; 3) of the conduction band.

3. What is the formula of dependence of metal resistance on temperature?
1) $R = R_0(1+\alpha_m t^\circ)$; 2) $R = AT^b e^{\frac{\Delta E}{2kT}}$; 3) $R = Ae^{\frac{\Delta E}{2kT}}$.

4. What is the formula of the dependence of resistance of semiconductors on temperature at T < 500 K?
1) $R = R_0(1+\alpha_m t^\circ)$; 2) $R = AT^b e^{\frac{\Delta E}{2kT}}$; 3) $R = Ae^{\frac{\Delta E}{2kT}}$.

5. Basic parts of the Wheatstone's bridge circuit are:
1) condenser; 2) slide-wire; 3) source of current;
4) thermistor; 5) galvanometer; 6) box of resistance.

6. The experimental (working) formula for determination of the thermistor resistance is:
1) $R_T = R_{box}\frac{R_1}{R_2}$; 2) $R_T = R_{box}\frac{l_1}{l_2}$; 3) $R_T = R_{box}\frac{R_2}{R_1}$.

7. Thermistor is:
1) termoresistence; 2) thermocouple; 3) semiconductor; 4) dielectric.

8. Give the way to find the temperature in the fist with help of thermistor.
1) place the thermistor in the hand, calculate the resistance of thermistor and substitute this value in the working formula;
2) place thermistor in the hand, calculate the resistance of thermistor and find temperature with help of the graph;
3) with help of thermometer.

9. The bridge circuit is considered balanced (or compensated) if:
1) the current in 2 "arms" of the bridge is absent;
2) the current in one "arm" of the bridge is absent;
3) the current in the diagonal of the bridge is absent;
4) the potentials on the ends of the bridge's diagonal are equal.

Test to laboratory work №22

Study of the work of the UHF – therapy device

1. What is the frequency range of UHF fluctuations?

1) 30 MHz - 300 MHz; **2)** 16 Hz - 20 kHz; **3)** 20 kHz - 1010 Hz.

2. What is the frequency of the electric field at UHF - therapy?

1) 15.2 MHz; **2)** 40.58 MHz; **3)** 200 MHz; **4)** 20 kHz.

3. Quantity of heat allocated per unit of volume per unit of time at electrolyte at UHF – therapy is determined under the formula:

1) $q = \frac{E^2}{\rho}$; **2)** $q = E^2_{эфф}\varepsilon\varepsilon_0\omega tq\delta$; **3)** $q = j^2\rho$; **4)** $q = k\frac{B^2_{эфф}\omega^2}{\rho}$.

4. Quantity of heat allocated per unit of dielectric volume per unit of time at UHF-therapy is determined under the formula:

1) $q = \frac{E^2}{\rho}$; **2)** $q = E^2_{эфф}\varepsilon\varepsilon_0\omega tq\delta$; **3)** $q = j^2\rho$; **4)** $q = k\frac{B^2_{эфф}\omega^2}{\rho}$.

5. Device of UHF - therapy is:

1) three-lamp generator; **2)** rectifier; **3)** duple-lamp generator.

6. What substance is heated faster at influence of UHF- field?

1) electrolyte; **2)** dielectric; **3)** electrolyte and dielectric are identical.

7. At resonance between therapeutic and anodic contour:

1) current in dipole aerial is maximal;
2) frequency in therapeutic contour equals to frequency in anodic contour;
3) the neon bulb between electrodes lights most brightly;
4) greatest power is allocated in therapeutic contour.

8. What is the purpose of the therapeutic contour?

1) for amplification of biological potentials;
2) for safety of patients;
3) for adjustment in resonance with the anodic contour;
4) for measurement of biopotentials on the surface of human body.

9. We do adjustment of the therapeutic contour in resonance, changing

1) R_C; **2)** L_T; **3)** C_A; **4)** C_T; **5)** L_C.

Test to laboratory work №23

Determination of a refractive index of substance with help of refractometer

1. Law of light refraction at transition from the medium1 to the medium 2 is defined by the formula:

1) $n_1=n_2\sin\alpha$; 2) $2d\sin\alpha=mk$; 3) $n_1=\frac{n_2}{\sin\alpha}$; 4) $\frac{\sin\alpha}{\sin\beta}=\frac{n_2}{n_1}$; 5) $I=I_0\cos^2\alpha$.

2. Law of light refraction at transition from the medium 1 to the medium 2 for the critical angle of refraction is determined by the formula:

1) $n_1=n_2\sin\alpha$; 2) $2d\sin\alpha=mk$; 3) $n_1=\frac{n_2}{\sin\alpha}$; 4) $\frac{\sin\alpha}{\sin\beta}=\frac{n_2}{n_1}$; 5) $I=I_0\cos^2\alpha$.

3. Absolute refractive index *n* of a medium is the ratio:

1) v/c; 2) λ_1/λ_2; 3) v_2/v_1; 4) n_2/n_1; 5) c/v.

4. Refractometer consists of:

1) eyepiece; 2) objective; 3) equalizer; 4) condenser; 5) visual tube; 6) lighting prism and measuring prism; 7) rotary prism.

5. Position of the border of "light and shade" in the eyepiece of the refractometer is defined by:

1) the refractive index of the researched liquid;
2) the critical angle of refraction; 3) the incidence angle.

6. Equalizer is necessary for:

1) creation of critical angle; 2) removing of dispersion of light;
3) removing of colour border of the "light and a shadow".

7. In the given work we study dependence:

1) $n = f(c)$; 2) $n = f(\alpha)$; 3) $n = f(\lambda)$; 4) $n = f(v)$.

8. Concentration of unknown liquid we can determine with help of:

1) working formula; 2) graph; 3) left scale; 4) right scale of the device.

9. The refractive index of a given medium *n* necessarily is:

1) less of refractive index of the glass of the measuring prisms;
2) equal to refractive index of the glass of the measuring prisms;
3) more of refractive index of the glass of the measuring prisms.

Test to laboratory work №24

Determination of concentration of sugar in a solution with help of polarimeter

1. Polarimetry (or saccharimetry) is the method of determination of the:
1) refractive index of the optically active substances;
2) wavelength of polarized light;
3) concentration of the optically active substances in a solution;
4) position of the plane of polarization of polarized light.

2. Point the ways of producing of the plane polarized light:
1) by reflection and refraction of light; 2) by scattering;
3) by absorption of light; 4) by double refraction.

3. Polarized is light wave:
1) in which fluctuations of vector **E** occurs in the single plane;
2) which has constant frequency; 3) which has constant wavelength;
4) in which fluctuations of vectors **E** and **B** are made in mutually perpendicular planes.

4. What are optically active substances?
1) substances that can decrease the surface tension coefficient;
2) substances capable to rotate the plane of polarization;
3) substances capable to increase the viscosity coefficient;
4) substances capable to produce plane polarized light.

5. The angle of rotation of a polarization plane depends on:
1) thickness of layer of substance; 2) concentration of substance;
3) wavelength of substance; 4) temperature of substance;
5) nature of substance.

6. Intensity of light after analyzer is equal to zero. Determine the angle between the principle planes of the polarizer and analyzer.
1) 0°; 2) 30°; 3) 90°; 4) 180°.

7. The filter in a polarimeter is established for:
1) amplification of the light intensity; 2) reception of monochromatic light;
3) decreasing of the light intensity; 4) rotation of the plane of polarization.

8. Formula for calculation of the specific rotation is:
1) $\alpha = \frac{\Delta\varphi}{Cl}$; 2) $\alpha = \frac{Cl}{\Delta\varphi}$; 3) $\Delta\varphi = \alpha Cl$; 4) $\alpha = \frac{\Delta\varphi}{C}$.

9. Point the formula of the Malus' law:
1) $I = I_0 \cos^2\alpha$; 2) dsinα = mk; 3) $\Delta\varphi = \alpha Cl$; 4) $E=E_0 \cos\varphi$.

Test to laboratory work №25
«Study of photoelectrocolorimeter»

1. Law of Buger-Lambert-Ber is equation:

1) $I = I_0 e^{-\chi cl}$; **2)** $I = I_0 \chi cl$; **3)** $I = \frac{I_0}{e^{-\chi cl}}$.

2. Transparency of solution (T) is determined by the formula:

1) $T = \frac{I}{I_0}$; **2)** $T = \lg \frac{I_0}{I}$; **3)** $T = \chi_\lambda cl$; **4)** $T = \frac{cl}{\chi_\lambda}$.

3. Optical density of substance (D) is determined by the formula:

1) $D = \frac{I}{I_0}$; **2)** $D = \lg \frac{I_0}{I}$; **3)** $D = \chi_\lambda cl$; **4)** $D = \frac{cl}{\chi_\lambda}$.

4. Optical density of substance (D) has dimension:

1) Kg / m^3; **2)** Pa·s; **3)** dimensionless size; **4)** W/m^2.

5. We plot the graph dependence of:

1) transparency **(T)** on concentration of the solution **(C)**;
2) transparency **(T)** on thickness of the solution **(l)**;
3) optical density **(D)** on concentration of the solution **(C)**;
4) optical density **(D)** on the thickness of the solution **(l)**.

6. For determination of D it is necessary to take the optical filter with:

1) the minimal value of D; **2)** the maximal value of D;
3) D=1; **4)** D=0.

7. What is the purpose of the galvanometer?

1) for equalizing of the photocurrents; **2)** for measurement of the current;
3) for measurement of the optical density; 4) for measurement of c.

8. Unknown concentration we found by means of:

1) working formula; **2)** graph; **3)** proportion; 4) measurement.

9. Galvanometer shows "0". It means that:

1) optical densities of compared solutions are equal;
2) optical densities of compared solutions are not equal;
3) device is not prepared to work;
4) device is prepared to work.

Test to laboratory work №26

FOCAL POWER DETERMINATION OF SPECTACLE LENSES BY MEANS OF DIOPTRIMETER

1. Focal power of a convex lens is 5 diopters. Optical force of this lens is
1) 1 m, 2) 5 m, 3) 0.5 m, 4) 0.2 m.

2. Reason of myopia is
1) reduced optical force of an eye, 2) shorted shape of an eyeball,
3) increased optical force of an eye, 4) extended shape of an eyeball.

3. The prism by force of 2 prismatic diopters (2 Δ) declines the image of the subject that is on the distance of 1 m towards vertex on
1) 1cm, 2) 2cm, 3) 3 cm, 4) 4 cm.

4. Measure of astigmatism is
1) value of light angle on the astigmatic lens,
2) degree of distortion of the given lens,
3) focal power difference in two main cross-sections,
4) difference of radiuses in two main cross-sections.

5. Reason of astigmatism in the human eye is
1) big incidence angle to the optical axis (astigmatism of skew fascicles),
2) disturbance of the spherical shape of a cornea,
3) irregular curvature of a lens.

6. If luminous points forms a circle, the researched lens is
1) spherical or prismatic, 2) astigmatic or prismatic,
3) spherical, 4) astigmatic.

7. If luminous points are parallel lines posed parallel to one of the main lens sections, the researched lens is
1) spherical or prismatic, 2) astigmatic or prismatic,
3) spherical, 4) astigmatic.

8. Number of divisions from the centre of the cross-hairs up to the centre of the dot grid defines number of
1) prismatic diopters, 2) spherical diopters,
3) astigmatic difference, 4) optical force of any lens.

9. It is possible to determine for astigmatic glasses on dioptrimeter
1) focal power, 2) astigmatic difference, 3) position of axis *(ax)* on the scale ТАБО, 4) prismatic activity.

10. What is the force of cylindrical element in diopters: sph – 0.5D, cyl – 5.0D, *ax* 10^0?
1) 0.5, 2) - 0.5, 3) 5, 4) – 5.

Test to laboratory work №27

Measurement of the size of small objects with the help of microscope

1. Linear magnification of the microscope with the photounite (N) is determined by the formula:

1) $N = \frac{L}{Cn}$; 2) $N=0.56\,LCn$; 3) $N = \frac{L}{0.56Cn}$.

2. Scale division value of the ocular grid is determined by the formula:

1) $\alpha = C_0Nm$; 2) $\alpha = \frac{CN}{m}$; 3) $\alpha = \frac{N}{Cm}$.

3. The increased size of the subject (L) and magnification (N) of microscope are known. What is the true size of the subject (l)?

1) $l = \frac{L^2}{N}$; 2) $l = LN$; 3) $l = \frac{N}{L}$; 4) $l = \frac{L}{N}$.

4. Give the 3 characteristics of the image received after ocular:

1) real; 2) virtual; 3) direct; 4) inverted; 5) increased; 6) decreased.

5. Point the 3 characteristics of the image received after objective:

1) real; 2) virtual; 3) direct; 4) inverted; 5) increased; 6) decreased.

6. Distinction limit (Z) of a microscope is:

1) the minimal angle of vision for which two next points of a subject are visible separately;
2) the minimal distance between two points of the subject which are visible separately;
3) the minimal distance between focuses of the objective and ocular;
4) ability of a microscope to give the separate image of fine details of a subject.

7. Four divisions (N=4) of the object - micrometer contains m = 20 divisions of ocular grid. The scale division value of the object - micrometer is C = 0.2 mm. Scale division value of the ocular grid is equal (mm):

1) 0.01; 2) 0.02; 3) 0.03; 4) 0.04.

8. Ways to reduce distinction limit of a microscope:

1) to increase wavelength of light λ and the refractive index of the medium n;
2) to decrease wavelength of light λ and the refractive index of the medium n;
3) to decrease λ, to increase the refractive index n;
4) to increase λ, to decrease the refractive index of the medium n.

9. Distinction limit of the microscope (Z) is determined by the formula:

1) $Z = \frac{0.5\lambda}{A}$; 2) $Z = \frac{0.61\lambda}{A}$; 3) $Z = \frac{0.61\lambda}{n\sin\alpha}$.

Test to laboratory work №28

Graduation of the spectroscope and determination of wavelength of spectral lines

1. Spectral analysis is method of:
1) determination of the chemical structure of substance on its spectrum;
2) determination of number of radiating (absorbing) atoms;
3) determination of the optical density of substance on its spectrum.

2. Basic kinds of spectrum are:
1) emission; 2) refraction and reflection; 3) linear; 4) absorption.

3. Linear spectra are typical for:
1) heated rigid bodies;
2) gases and steams of metals in atomic condition;
3) steams and gases in the molecular condition.

4. Continuous spectra are typical for:
1) heated solid, liquid bodies and gases at high pressure;
2) gases and steams of metals in atomic condition;
3) steams and gases in the molecular condition.

5. Graduation curve of the spectroscope is dependence:
1) $L=f(\lambda)$; 2) $D=f(\lambda)$; 3) $I=f(\lambda)$.

6. Spectrum of the $KMNO_4$ consists of:
1) 1 red line; 2) several dark lines on continuous spectrum;
3) several yellow lines; 4) four color lines of linear spectrum.

7. Spectrum of Na consists of:
1) 1 bright red line; 2) 1 bright yellow line;
3) 1 dark strip; 4) several dark lines.

8. In the given work we observed the following spectra:
1) absorption; 2) emission; 3) linear;
4) continuous; 5) band.

9. On a scale of the screw (horizontal) 7 divisions are opened. On the scale of drum (vertical) are 47 divisions. Find the indications of reading off device of spectroscope (mm):
1) 7.47
2) 7.94
3) 6.47
4) 6.94

Test to laboratory work №29

Study of the work of the gaseous optical laser

1. **Laser radiation is**
   1) thermal; 2) spontaneous; 3) induced; 4) white.

2. **Working medium of the given gaseous laser are the atoms of:**
   1) helium ; 2) neon; 3) mix of He and Ne; 4) mercury.

3. **Transition of atoms in the gaseous laser from the excited level 3 to the basic level 1 happens:**
   1) at impact with other exited atom;
   2) at impact with nonexited atoms;
   3) due to increase of internal energy;
   4) under operation of external quantum.

4. **Each laser must have:**
   1) discharge tube; 2) two mirrors; 3) active medium;
   4) pump; 5) resonator.

5. **Diffraction grating is used for determination of:**
   1) laser wavelength; 2) size of licopodium particles;
   3) divergence of laser beam.

6. **For determination of wavelength of laser we must know:**
   1) X, L, k; 2) x, L; 3) d, X, L, k; 4) X, L, k, sin α.

7. **For determination of divergence of laser beam we measured:**
   1) D_1, D_2, λ; 2) D_1, D_2, L; 3) D, X, L, k; 4) X, L, k, sin α.

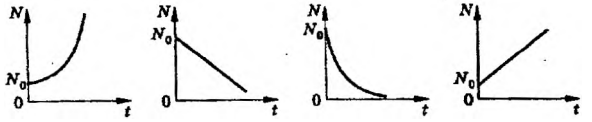
8. **Key properties of laser radiation are:**
   1) coherency; 2) monochromaticity; 3) polarization;
   4) small divergence (narrowness); 5) big power and intensity.

9. **On what property of laser radiation does hologram base?**
   1) coherency; 2) monochromaticity; 3) polarization;
   4) small divergence (narrowness); 5) big power and intensity.

Test to laboratory work №30

USE AND APPLICATION OF THE DOSIMETER-RADIOMETER АНРИ–01–02 "PINE (COCHA)" FOR RADIATION CONTROL

1. Specify the graph of the basic law of radioactive decay



1) 2) 3) 4)

2. Letter N in the formula of the basic law of radioactive decay is:

1) initial number of nucleus; 2) number of the disintegrated nucleus ;
3) number of the nucleus disintegrating per unit of time;
4) number of not disintegrated nucleus.

3. Activity of a nuclear tracer is:

1) decay energy of nucleus ; 2) decay rate;
3) probability of decay of the radioactive nucleus;
4) time during which half of nucleus disintegrates.

4. Activity unit in SI:

1) Sievert; 2) Gray; 3) Curie; 4) Becquerel.

5. Specify the correct correspondence of doses and unities of their measurement:

A) Absorbed	a) Sievert
B) Exposition	b) Q / kg (Roentgen)
C) Equivalent	c) Gray

1)Aa, ba, Cc; 2)Ac, Bc, Ca; 3) Ac, Bb, Ca; 4) Ac, Bb, Cc.

6. In our work dosimeter can measure:

1) power of air (exposition) dose; 2) power of absorbed dose;
3) radiant (flux) density of β - radiation; 4) activity of radionuclides.

7. At increase of distance from a radioactive source, power of the equivalent dose

1) decreases proportionally to distance;
2) increases proportionally to distance;
3) decreases proportionally to the square of distance;
4) increases proportionally to the square of distance.

8. Specify value of the norm of the power of air dose of natural radioactive background (mcR/hour) and the limiting admissible equivalent annual dose for adult population (ber/year):

1) 10-20, 0.5; 2) 5-10, 0.1; 3) 20-30, 1; 4) 30-40, 1.

9. Basic methods of protection against an ionizing radiation are:

1) protection by time; 2) personal protective equipment;
3) protection by a material; 4) protection in distance

TABLES

SOME PHYSICAL CONSTANTS

Constant	Symbol	Value
Velocity of light in vacuum	c	$3 \cdot 10^8$ m/s
Permeability of free space	μ_0	$4\pi \cdot 10^{-7}$ H/m
Permittivity of free space	ε_0	$8.85 \cdot 10^{-12}$ F/m
Universal constant of gravitation	G	$6.67 \cdot 10^{-11}$ Nm^2/kg^2
Planck constant	h	$6.63 \cdot 10^{-34}$ J·s
Rest mass of electron	m_e	$9.1 \cdot 10^{-31}$ kg
Rest mass of proton	m_p	$1.673 \cdot 10^{-27}$ kg
Rest mass of neutron	m_n	$1.674 \cdot 10^{-27}$ kg
Electron charge	e	$1.6 \cdot 10^{-19}$ C
Specific charge of electron	e/m	$1.76 \cdot 10^{11}$ C/kg
Atomic mass unit	u	$1.66 \cdot 10^{-27}$ kg
Avogadro constant	N_A	$6.02 \cdot 10^{23}$ mol^{-1}
Faraday constant	F	$9.65 \cdot 10^4$ C/mol
Molar gas constant	R	8.31 J/(mol·K)
Boltzmann constant	$k=R/N_A$	$1.38 \cdot 10^{-23}$ J/K
Acceleration due to gravity	g	9.81 m/s^2
Stefan constant	σ	$5.7 \cdot 10^{-8}$ $W/(m^2 \cdot K^4)$
Bohr magneton	μ_B	$9.27 \cdot 10^{-24}$ J/T
Coefficient in the law of Coulomb	$k=1/(4\pi\varepsilon_0)$	$9.00 \cdot 10^9$ m/F
Absolute zero		- 273.16 °C

Units in Physics

Quantity	*Unit*	*Symbol*
length	metre	m
mass	kilogram	kg
atomic mass	atomic mass unit	u
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
luminous intensity	candela	cd
amount of substance	mole	mol
frequency	hertz	Hz
force	newton	N
pressure and stress	pascal	Pa
work, energy, heat	joule	J
power	watt	W
electric charge	coulomb	C
electric potential difference	volt	V
electromotive force	volt	V
electric resistance	ohm	Ω
electric conductance	Siemens	S
electric capacitance	farad	F
magnetic flux	weber	W
magnetic flux density (magnetic induction)	tesla	T
inductance	henry	H
luminous flux	lumen	lm
illuminance	lux	lx
activity (of radioactive source)	becquerel	Bq
specific heat capacity	J/(kg·K)	*c*
momentum	N·s	*P*
moment of a force	N·m	*M*
torque	N·m	*T*
electrical resistivity	Ω·m	ρ
electrical conductivity	S/m	σ
current density	A/m^2	j
permittivity	F/m	ε
electric field strength	N/C or V/m	*E*
capacitance	F	*C*
permeability	H/M	μ
moment of inertia	$kg·m^2$	I
Young modulus	Pa	*E*
surface tension	N/m	α
viscosity	Pa·s	η
thermal conductivity	W/(m·K)	*k*

Student t Distribution

Degrees of freedom f = n-1	Significance level α,%		
	2.5 (one tail)	0.5 (one tail)	0.05 (one tail)
	5 (two tails)	1 (two tails)	0.1 (two tails)
1	12.71	63.66	64.60
2	4.30	9.92	31.6
3	3.18	5.84	12.92
4	2.78	4.60	8.61
5	2.57	4.03	6.87
6	2.45	3.71	5.96
7	2.37	3.50	5.41
8	2.31	3.36	5.04
9	2.26	3.25	4.78
10	2.23	3.17	4.59
11	2.20	3.11	4.44
12	2.18	3.05	4.32
13	2.16	3.01	4.22
14	2.14	2.98	4.14
15	2.13	2.95	4.07
16	2.12	2.92	4.02
17	2.11	2.90	3.97
18	2.10	2.88	3.92
19	2.09	2.86	3.88
20	2.09	2.85	3.85
21	2.08	2.83	3.82
22	2.07	2.82	3.79
23	2.07	2.81	3.77
24	2.06	2.80	3.75
25	2.06	2.79	3.73
26	2.06	2.78	3.71
27	2.05	2.77	3.69
28	2.05	2.76	3.67
29	2.05	2.76	3.66
30	2.04	2.75	3.65
40	2.02	2.70	3.55
60	2.0	2.66	3.46
120	1.98	2.62	3.37
∞	1.96	2.58	3.29

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